Lec 8, 2/9/09

Spanning sets can be too big to be linearly independent. Linearly independent sets can be two small to span. Bases are just the right size. More precisely:

Theorem: Let V be a (finite dimensional) vector space.
(i) Every spanning set of V contains a basis for V.
(ii) Every linearly independent set in V is contained in a basis for V.

Outline of Proof:
(i) Let S be a spanning set for V. If S is linearly independent, then S is a basis. If not, some vector is a linear combination of the others, and this vector can be removed with the resulting subset being a spanning set. Continue until a linearly independent spanning set is obtained.

(ii) Let S be a linearly independent subset of V. If S spans V, then S is a basis. If not, there is a vector $\bar{v}$ in V that is not in $sp(S)$, and then $S \cup \{\bar{v}\}$ is also linearly independent. Continue until a spanning set is obtained. $\square$

Note: The theorem is also true if V is infinite dimensional.

Corollary If $\dim V = n$, then
(i) any subset of V with more than n vectors is linearly dependent, and
(ii) any subset of V with less than n vectors cannot span V.