(26)

We now have two methods for finding a basis for a subspace W of $\mathbb{R}^\ell$, given a spanning set for W.
Let $W = sp\{\overline{w}_1, \dots, \overline{w}_n\} \subseteq \mathbb{R}^\ell$.

<u>Method 1</u>

① Form a matrix A with $\overline{w}_i$ as <u>rows</u>
② Find the row echelon form E of A.
③ Nonzero rows of E then form a basis for W.

This method is best for finding a "nice" basis for W.

<u>Method 2</u>:

① Form a matrix A with $\overline{w}_i$ as <u>columns</u>
② Find the row echelon form E of A
③ Columns of A (the $\overline{w}_i$) corresponding to columns of E with pivots form a basis for W.

This method is best for finding a basis for W that is a <u>subset</u> of the spanning set.

We can generalize this to other vector spaces by "coordinatizing"

<u>Example</u>: Find a basis for the subspace W of $P_2(t)$ spanned by $S = \{1+2t-t^2, -2-4t+2t^2, 2+3t+t^2, 3+5t\}$.

Note that $a+bt+ct^2$ is in $sp(S)$ if and only if (a,b,c) is in the span of $\{(1,2,-1), (-2,-4,2), (2,3,1), (3,5,0)\} \subseteq \mathbb{R}^3$.

<u>Method 1</u>: $A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & -4 & 2 \\ 2 & 3 & 1 \\ 3 & 5 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & -1 & 3 \\ 0 & -1 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = E.$

So a basis for W is $\{1+2t-t^2, t-3t^2\}$.

<u>Method 2</u>: $A = \begin{array}{c} \begin{array}{cccc} * & & * & \end{array} \\ \begin{bmatrix} 1 & -2 & 2 & 3 \\ 2 & -4 & 3 & 5 \\ -1 & 2 & 1 & 0 \end{bmatrix} \end{array} \to \begin{bmatrix} 1 & -2 & 2 & 3 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 3 & 3 \end{bmatrix} \to \begin{array}{c} \begin{array}{cccc} * & & * & \end{array} \\ \begin{bmatrix} \boxed{1} & -2 & 2 & 3 \\ 0 & 0 & \boxed{1} & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$

So a basis for W is $\{1+2t-t^2, 2+3t+t^2\} \subseteq S$.