(28)

<u>EXAMPLE</u>: Let $A = \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ 2 & 1 & 4 & 2 & 7 \\ 1 & 1 & 3 & 2 & 6 \\ 4 & 2 & 8 & 5 & 15 \end{bmatrix}$ as in the example on p25.

Thus A is the coefficient matrix of the system

$$\begin{aligned} x_1 \quad + x_3 + x_4 + 2x_5 &= 0 \\ 2x_1 + x_2 + 4x_3 + 2x_4 + 7x_5 &= 0 \\ x_1 + x_2 + 3x_3 + 2x_4 + 6x_5 &= 0 \\ 4x_1 + 2x_2 + 8x_3 + 5x_4 + 15x_5 &= 0. \end{aligned}$$

We saw before that the echelon form is $E = \begin{bmatrix} \boxed{1} & 0 & 1 & 1 & 2 \\ 0 & \boxed{1} & 2 & 0 & 3 \\ 0 & 0 & 0 & \boxed{1} & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$,

corresponding to

$$\begin{aligned} x_1 \quad + x_3 + x_4 + 2x_5 &= 0 \\ x_2 + 2x_3 \quad + 3x_5 &= 0 \\ x_4 + x_5 &= 0 \end{aligned}$$

Thus x_1, x_2, x_4 are pivot variables, and <u>$\dim \mathcal{C}(A) = 3$</u>.
The free variables are x_3, x_5, and <u>$\dim \mathcal{N}(A) = 2$</u>.

Solving, we have

$$\begin{aligned} x_4 &= -x_5 \\ x_2 &= -2x_3 - 3x_5 \\ x_1 &= -x_3 - x_4 - 2x_5 = -x_3 + x_5 - 2x_5 = -x_3 - x_5. \end{aligned}$$

Letting $x_3 = 1$, $x_5 = 0$ we get the basis vector $\begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$.

Letting $x_3 = 0$, $x_5 = 1$ we obtain $\begin{bmatrix} -1 \\ -3 \\ 0 \\ -1 \\ 1 \end{bmatrix}$

Hence $\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$ is a <u>basis</u> for $\mathcal{N}(A)$.

Substituting $x_3 = \alpha$, $x_4 = \beta$, we have

$$\mathcal{N}(A) = \left\{ \begin{bmatrix} -\alpha - \beta \\ -2\alpha - 3\beta \\ \alpha \\ -\beta \\ \beta \end{bmatrix} : \alpha, \beta \in \mathbb{R} \right\} = \left\{ \alpha \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} -1 \\ -3 \\ 0 \\ -1 \\ 1 \end{bmatrix} : \alpha, \beta \in \mathbb{R} \right\}.$$

End Lec 9