Lec 2 1/23/09

Basic Examples

① $\mathbb{R}^n = \{(a_1, a_2, \cdots, a_n) \mid a_i \in \mathbb{R}\}$ is a vector space over $\mathbb{R}$ with

$$(a_1, a_2, \dots, a_n) + (b_1, b_2, \dots, b_n) = (a_1+b_1, \cdots, a_n+b_n)$$
$$\alpha(a_1, \dots, a_n) = (\alpha a_1, \cdots, \alpha a_n)$$

All axioms are easily verified using well-known properties of real numbers [See Theorem 1.1, Theorem 2.1, Problem 2.3].

Observe that $\bar{0} = (0, 0, \cdots, 0)$ and if $\bar{v} = (a_1, \cdots, a_n)$, then $-v = (-a_1, \dots, -a_n)$.

[Verify this!]

Notes: ⓘ We could define K^n for any field K - simply replace "$\mathbb{R}$" above with "K." (For example, $\mathbb{Q}^n$, $\mathbb{C}^n$).

ⓘⓘ Vectors in $\mathbb{R}^n$ are denoted as row vectors in the text. It is sometimes convenient to consider $\mathbb{R}^n$ as a set of column vectors. We will use $\mathbb{R}^n_{row}$, $\mathbb{R}^n_{col}$ to distinguish between the two notations if there is a chance of ambiguity.

② $M_{m,n}(\mathbb{R})$ = set of all $m \times n$ matrices with real number entries is a vector space over $\mathbb{R}$ with the usual (componentwise) matrix addition and scalar multiplication [See Theorem 2.1].

This is a generalization of Example ① above:

$$M_{1,n}(\mathbb{R}) = \mathbb{R}^n_{col}, \quad M_{n,1}(\mathbb{R}) = \mathbb{R}^n_{row} = \mathbb{R}^n.$$

We also write $M_n(\mathbb{R})$ for $M_{n,n}(\mathbb{R})$.
Again, any field K may be substituted for $\mathbb{R}$.