<u>Proof:</u> Let $B_U = \{\bar{u}_1, \dots, \bar{u}_m\}$ be a basis for U. We can extend B_U to a basis $B = \{\bar{u}_1, \dots, \bar{u}_m, \bar{w}_1, \dots, \bar{w}_n\}$ for V. Let $W = sp\{\bar{w}_1, \dots, \bar{w}_n\}$ (so $\{\bar{w}_1, \dots, \bar{w}_n\}$ is a basis for W). We will show $V = U \oplus W$.

Let $\bar{v}$ be any vector in V. Since B spans V, we can write $\bar{v} = \alpha_1 \bar{u}_1 + \dots + \alpha_m u_m + \beta_1 w_1 + \dots + \beta_n w_n$ for some scalars α_i, β_j. Since U, W are subspaces of V, $\bar{u} = \alpha_1 \bar{u}_1 + \dots + \alpha_m \bar{u}_m \in U$ and $\bar{w} = \beta_1 \bar{w}_1 + \dots + \beta_n \bar{w}_n \in W$. Hence $\bar{v} = \bar{u} + \bar{w} \in U + W$, and so $\underline{V = U + W}$.

By HW#3 Problem 4.94,

$$\underline{U \cap W} = sp\{\bar{u}_1, \dots, \bar{u}_m\} \cap sp\{\bar{w}_1, \dots, \bar{w}_n\} = \underline{\{\bar{0}\}},$$

and so $V = U \oplus W$. ∎

<u>Coordinate Vectors</u> (§4.11)

Recall that if V is a vector space with basis $B = \{\bar{v}_1, \dots, \bar{v}_n\}$, then every vector $\bar{v} \in V$ can be written in a <u>unique</u> way in the form $\bar{v} = \alpha_1 \bar{v}_1 + \dots + \alpha_n \bar{v}_n$.

<u>Defn:</u> Let $V, B, \bar{v}$ be as above. The <u>coordinate vector of $\bar{v}$ relative to B</u> is

$$[\bar{v}]_B = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} \quad (\text{or } [\alpha_1, \alpha_2, \dots, \alpha_n])$$

<u>Notes:</u> ① The basis must be thought of as an <u>ordered</u> set in this context. Changing the order of B will result in a different coordinate vector.

② $\bar{v} \longleftrightarrow [\bar{v}]_B$ gives a bijection between V and K^n.
Also, this correspondence "respects" the operations:

(i) $[\bar{v} + \bar{w}]_B = [\bar{v}]_B + [\bar{w}]_B$ for all $\bar{v}, \bar{w} \in V$

(ii) if $\alpha \in K$, $\bar{v} \in V$ then $[\alpha \bar{v}]_B = \alpha [\bar{v}]_B$.