(36)

(EXAMPLES, cont.)

④ Let V be any finite dimensional vector space over $\mathbb{R}$, with basis $B = \{\bar{v}_1, \dots, \bar{v}_n\}$. We can define an inner product on V by

$$\langle \bar{u}, \bar{v} \rangle = [\bar{u}]_B \cdot [\bar{v}]_B$$

That is, if $\bar{u} = \alpha_1 \bar{v}_1 + \cdots + \alpha_n \bar{v}_n$, $\bar{v} = \beta_1 v_1 + \cdots + \beta_n \bar{v}_n$, then

$$\langle \bar{u}, \bar{v} \rangle = \alpha_1\beta_1 + \cdots + \alpha_n\beta_n.$$

[Of course, this inner product depends on the chosen basis.]

⑤ Let $V = P_n(t)$. For $f(t), g(t) \in V$, define

$$\langle f(t), g(t) \rangle = \int_0^1 f(t)g(t)\,dt$$

We verify that this is an inner product.

(I_1) $\langle f(t)+g(t), h(t) \rangle = \int_0^1 (f(t)+g(t))h(t)\,dt = \int_0^1 f(t)h(t) + g(t)h(t)\,dt$

$$= \int_0^1 f(t)h(t)\,dt + \int_0^1 g(t)h(t)\,dt = \langle f(t), h(t) \rangle + \langle g(t), h(t) \rangle$$

$$\langle \alpha f(t), g(t) \rangle = \int_0^1 \alpha f(t)g(t)\,dt = \alpha \int_0^1 f(t)g(t)\,dt = \alpha \langle f(t), g(t) \rangle.$$

(I_2) $\langle f(t), g(t) \rangle = \int_0^1 f(t)g(t)\,dt = \int_0^1 g(t)f(t)\,dt = \langle g(t), f(t) \rangle.$

(I_3) $\langle f(t), f(t) \rangle = \int_0^1 f(t)^2\,dt \geq 0$ since $f(t)^2 \geq 0$.

Also, the integral of a nonnegative continuous function is 0 if and only if the function is 0 so $\langle f(t), f(t) \rangle = 0$ if and only if $f(t) = 0$. □

End Lec 12

Lec 13, 2/20/09

Norm of a Vector

Defn: Let V be an inner product space. The norm or length of a vector $\bar{v}$ in V, denoted $\|\bar{v}\|$, is defined to be

$$\|\bar{v}\| = \sqrt{\langle \bar{v}, \bar{v} \rangle}.$$

[Observe also that $\langle \bar{v}, \bar{v} \rangle = \|\bar{v}\|^2$.]