Remarks ① In $\mathbb{R}^n$ with the standard inner product,
$$\|(a_1, a_2, \cdots, a_n)\| = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}.$$
In $\mathbb{R}^2$ and $\mathbb{R}^3$, this coincides with the usual idea of "length" of the vector by the Pythagorean theorem.

② the length of a vector in an inner product space V depends completely on the inner product defined on V.

Some Properties of the Norm (§7.4)

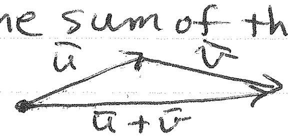
Theorem (Cauchy-Schwarz Inequality) If V is a real inner product space, then
$$|\langle \bar{u}, \bar{v} \rangle| \leq \|\bar{u}\| \|\bar{v}\|$$
for all $\bar{u}, \bar{v}$ in V.

Proof: See text [Problem 7.8]. $\square$

We also have the following basic properties, the third of which requires the Cauchy-Schwarz Inequality.

theorem: If V is a real inner product space, the norm satisfies:
[N1] $\|\bar{v}\| \geq 0$ for all $\bar{v} \in V$ and $\|\bar{v}\| = 0$ if and only if $\bar{v} = \bar{0}$.
[N2] $\|\alpha \bar{v}\| = |\alpha| \|\bar{v}\|$ for all $\alpha \in \mathbb{R}$, $\bar{v} \in V$.
[N3] $\|\bar{u} + \bar{v}\| \leq \|\bar{u}\| + \|\bar{v}\|$ for all $\bar{u}, \bar{v} \in V$ [Triangle Inequality]

Remarks ① In $\mathbb{R}^2$ and $\mathbb{R}^3$, the Triangle Inequality says that the length of one side of a triangle is less than or equal to the sum of the lengths of the other two sides.



② If $\bar{v}$ is any nonzero vector, then [N2] implies that $\hat{v} = \frac{1}{\|\bar{v}\|} \bar{v}$ is a unit vector (i.e. length 1) in the direction of $\bar{v}$.