 Lec15, 2/27/09

Corollary: If W is a subspace of a finite dimensional inner product space V, then $\dim V = \dim W + \dim W^\perp$.

Corollary: If V is a finite dimensional inner-product space and W is a subspace of V, then $(W^\perp)^\perp = W$.

Proof: By Remark (3) above, $W \subseteq (W^\perp)^\perp$. We will show equality by showing $\dim W = \dim (W^\perp)^\perp$. (Since $(W^\perp)^\perp$ is finite dimensional, this implies $W = (W^\perp)^\perp$.)

By the Corollary above,

$$\dim V = \dim W + \dim W^\perp$$

and $$\dim V = \dim W^\perp + \dim (W^\perp)^\perp.$$

Hence $\dim W = \dim (W^\perp)^\perp$ and so $W = (W^\perp)^\perp$. $\square$

Orthogonal Sets (§7.6)

Defn: Let V be an inner product space.
A set S of vectors in V is an orthogonal set if $\langle \bar{u}, \bar{v} \rangle = 0$ for all $\bar{u} \neq \bar{v}$ in S. An orthogonal set S is orthonormal if $\|\bar{u}\| = 1$ for all $u \in S$.

Remark: If $S = \{\bar{v}_1, \dots, \bar{v}_n\}$ is orthogonal, then for any scalars $\alpha_1, \dots, \alpha_n$ the set $S' = \{\alpha_1 \bar{v}_1, \dots, \alpha_n \bar{v}_n\}$ is also orthogonal (HW problem). Thus if no $\bar{v}_i$ is $\bar{0}$, then the set $\{\frac{1}{\|\bar{v}_1\|}\bar{v}_1, \dots, \frac{1}{\|\bar{v}_n\|}\bar{v}_n\}$ is an orthonormal set.

Theorem: If S is an orthogonal set of nonzero vectors, then S is linearly independent.

Proof: Let $\bar{v}_1, \dots, \bar{v}_n$ be vectors in S and suppose

$$\alpha_1 \bar{v}_1 + \dots + \alpha_n \bar{v}_n = \bar{0}.$$

[continued →]