EXAMPLES:

① Let $V=\mathbb{R}^3$, $E=\{\bar{e}_1,\bar{e}_2,\bar{e}_3\}$, $\bar{e}_1=(1,0,0)$, $\bar{e}_2=(0,1,0)$, $\bar{e}_3=(0,0,1)$, and $B=\{\bar{v}_1,\bar{v}_2,\bar{v}_3\}$, $\bar{v}_1=(1,1,0)$, $\bar{v}_2=(1,0,1)$, $\bar{v}_3=(0,1,1)$.

P = change of basis matrix from E to B:

Write the new basis vectors (B) in terms of the old (E).

$\bar{v}_1=(1,1,0)=1e_1+1e_2+0e_3$
$\bar{v}_2=(1,0,1)=1e_1+0e_2+1e_3$ so $[\bar{v}_1]_E=\begin{bmatrix}1\\1\\0\end{bmatrix}$, $[\bar{v}_2]_E=\begin{bmatrix}1\\0\\1\end{bmatrix}$, $[\bar{v}_3]_E=\begin{bmatrix}0\\1\\1\end{bmatrix}$,
$\bar{v}_3=(0,1,1)=0e_1+1e_2+1e_3$

hence $\boxed{P=\begin{bmatrix}1&1&0\\1&0&1\\0&1&1\end{bmatrix}}$

[Note: The basis change matrix from the standard basis E for $\mathbb{R}^n$ to any basis B for $\mathbb{R}^n$ will have the vectors in B as columns.]

Let $\bar{w}=3\bar{v}_1+4\bar{v}_2+5\bar{v}_3$, so that $[\bar{w}]_B=\begin{bmatrix}3\\4\\5\end{bmatrix}$. We have

$P[\bar{w}]_B=\begin{bmatrix}1&1&0\\1&0&1\\0&1&1\end{bmatrix}\begin{bmatrix}3\\4\\5\end{bmatrix}=\begin{bmatrix}7\\8\\9\end{bmatrix}$. Observe that

$\bar{w}=3(1,1,0)+4(1,0,1)+5(0,1,1)=(7,8,9)$ and so $[\bar{w}]_E=\begin{bmatrix}7\\8\\9\end{bmatrix}$.

End Lec 21 Hence $P[\bar{w}]_B=[\bar{w}]_E$ as expected.

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Q = change of basis matrix from B to E:

First, the coordinates of (a,b,c) relative to B are x,y,z, where $x(1,1,0)+y(1,0,1)+z(0,1,1)=(a,b,c)$.

Setting up the system of equations and solving, we obtain

$x=\frac{1}{2}a+\frac{1}{2}b-\frac{1}{2}c$, $y=\frac{1}{2}a-\frac{1}{2}b+\frac{1}{2}c$, $z=-\frac{1}{2}a+\frac{1}{2}b+\frac{1}{2}c$.

In particular, $[\bar{e}_1]_B=\begin{bmatrix}1/2\\1/2\\-1/2\end{bmatrix}$; $[\bar{e}_2]_B=\begin{bmatrix}1/2\\-1/2\\1/2\end{bmatrix}$; $[\bar{e}_3]_B=\begin{bmatrix}-1/2\\1/2\\1/2\end{bmatrix}$; thus $\boxed{Q=\frac{1}{2}\begin{bmatrix}1&1&-1\\1&-1&1\\-1&1&1\end{bmatrix}}$

[Check that $Q=P^{-1}$.]

If $\bar{w}=(-2,6,4)=-2\bar{e}_1+6\bar{e}_2+4\bar{e}_3$, so $[\bar{w}]_E=\begin{bmatrix}-2\\6\\4\end{bmatrix}$, then

$[\bar{w}]_B=Q[\bar{w}]_E=\frac{1}{2}\begin{bmatrix}1&1&-1\\1&-1&1\\-1&1&1\end{bmatrix}\begin{bmatrix}-2\\6\\4\end{bmatrix}=\begin{bmatrix}0\\-2\\6\end{bmatrix}$. Check that $\bar{w}=0\bar{v}_1-2\bar{v}_2+6\bar{v}_3$. □