Lec 23 3/18/09

Characteristic Polynomial (§9.3)

We have seen that there is a correspondence between matrices and linear operators:

$$A \text{ an } n\times n \text{ matrix} \longleftrightarrow \varphi_A : K^n \to K^n$$

$$T: V \to V \text{ linear operator} \longleftrightarrow [T]_B, \ B \text{ a basis for } V.$$

The definitions and results in what follows can be stated in either context. We will use whichever is more convenient. [See § 9.1]

If A is a square matrix, then for any positive integer k, the product $A^k = \underbrace{A \cdot A \cdots A}_{k}$ is defined.

If $f(t) = a_n t^n + \cdots + a_1 t + a_0$ is a polynomial over K, define

$$f(A) = a_n A^n + \cdots + a_1 A + a_0 I.$$

See Theorem 9.1 for several properties of this evaluation. [See § 9.2 for other details.]

Associated with each $n\times n$ matrix, there is a particular monic polynomial of degree n:

Defn: Let A be an $n\times n$ matrix. The characteristic polynomial of A is

$$\det(tI_n - A) = \Delta_A(t)$$

Thus, if $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$ then

$$\Delta_A(t) = \begin{vmatrix} t-a_{11} & -a_{12} & \cdots & -a_{1n} \\ -a_{21} & t-a_{22} & \cdots & -a_{2n} \\ \vdots & \vdots & & \vdots \\ -a_{n1} & -a_{n2} & \cdots & t-a_{nn} \end{vmatrix}.$$