(EXAMPLES, cont.)

③ Block triangular matrix:
(square blocks along diagonal, zeros below, anything above)

$$A = \left[\begin{array}{cc:cc} 1 & -2 & 5 & 6 \\ 4 & 5 & 3 & -12 \\ \hdashline 0 & 0 & 2 & -3 \\ 0 & 0 & -6 & 1 \end{array}\right]$$

$$\Delta_A(t) = \left|\begin{array}{cc:cc} t-1 & 2 & -5 & -6 \\ -4 & t-5 & -3 & 12 \\ \hdashline 0 & 0 & t-2 & 3 \\ 0 & 0 & 6 & t-1 \end{array}\right| = \begin{vmatrix} t-1 & 2 \\ -4 & t-5 \end{vmatrix} \cdot \begin{vmatrix} t-2 & 3 \\ 6 & t-1 \end{vmatrix}$$ (= Product of determinants of blocks)

$$= [(t-1)(t-5)+8][(t-2)(t-1)-18] = (t^2-6t+5+8)(t^2-3t+2-18)$$

$$= \boxed{(t^2-6t+13)(t^2-3t-16) = \Delta_A(t)}$$

End Lec 23

Lec 24 3/20/09 The following result allows us to define the characteristic polynomial of a linear operator.

<u>Theorem</u> If A and B are similar matrices, then $\Delta_A(t) = \Delta_B(t)$.

<u>Proof</u>: Let $B = P^{-1}AP$. We have

$$\Delta_B(t) = \det(tI - B) = \det(tI - P^{-1}AP) = \det(P^{-1}tIP - P^{-1}AP)$$
$$= \det(P^{-1}(tI-A)P) = \det(tI - A) = \Delta_A(t),$$

since similar matrices have the same determinant. ∎

<u>Defn</u>: Let V be a finite dimensional vector space, B any basis for V, and $T: V \to V$ a linear operator. Denote $A = [T]_B$. The <u>characteristic polynomial</u> of T, denoted $\Delta_T(t)$, is $\Delta_A(t)$.

<u>Note</u>: If B' is another basis for V, then $A' = [T]_{B'}$ is similar to A, so $\Delta_{A'}(t) = \Delta_A(t)$. Hence $\Delta_T(t)$ does <u>not</u> depend on the basis chosen.