<u>Corollary</u>: Let $V$ be a vector space over $K$.
If $\alpha \in K$, $\bar{v} \in V$, then $(-\alpha)\bar{v} = -(\alpha\bar{v}) = \alpha(-\bar{v})$.

<u>Proof</u>: This follows from the previous corollary, along with axiom [M3] and various properties of scalars.
For example: $(-\alpha)\bar{v} = ((-1)\alpha)\bar{v} = (-1)(\alpha\bar{v})$ by [M3]
$= -(\alpha\bar{v})$ by the corollary.
[<u>Exercise</u>: Complete the proof.] □

<u>Note</u>: The Corollary above is proved in a different way in the text. [See Problem 4.2].

End Lec 2]

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Lec 3 1/26/09 We also have the following important converse to the theorem:

<u>Theorem</u>: Let $V$ be a vector space over $K$.
If $\alpha\bar{v} = \bar{0}$ for $\alpha \in K$, $\bar{v} \in V$, then either $\alpha = 0$ or $\bar{v} = \bar{0}$.

<u>Proof</u>: [See also Probem 4.2 in text.]
Suppose $\alpha\bar{v} = \bar{0}$, but $\alpha \neq 0$. We need to show $\bar{v} = \bar{0}$.
Since $\alpha \neq 0$, there is a multiplicative inverse $\alpha^{-1}$ of $\alpha$ in $K$, so that $\alpha^{-1}\alpha = 1$. We then have

$$\begin{aligned}
\bar{0} &= \alpha^{-1}\bar{0} && \text{by the Theorem} \\
&= \alpha^{-1}(\alpha\bar{v}) && \text{since } \alpha\bar{v} = \bar{0} \\
&= (\alpha^{-1}\alpha)\bar{v} && \text{by axiom [M3]} \\
&= 1\bar{v} \\
&= \bar{v} && \text{by axiom [M4]}.
\end{aligned}$$

Hence $\bar{v} = \bar{0}$, as claimed. □