The results suggested by the examples above are:

Theorem: Let A be an $n\times n$ matrix. If $\Delta_A(t)=t^n+a_{n-1}t^{n-1}+\cdots+a_1t+a_0$, then
$$\boxed{\begin{aligned} a_{n-1} &= -\mathrm{tr}(A) \\ a_0 &= (-1)^n\det(A)\end{aligned}}$$

Sketch of Proof: First, $a_0=\Delta_A(0)=\det(0I-A)=\det(-A)=(-1)^n\det A$. Finding a_{n-1} is a little trickier - consider where the factors with t^{n-1} occur in $\det(tI-A)$. □

Theorem: (Cayley-Hamilton Theorem) If A is a square matrix with characteristic polynomial $\Delta_A(t)$, then $\Delta_A(A)=0$.

Remarks: Recall that if $F,G: V\to V$ are linear operators, then so are $F+G$, αF, and $F\circ G$, and $[F+G]_B=[F]_B+[G]_B$, $[\alpha F]_B=\alpha[F]_B$ and $[F\circ G]_B=[F]_B[G]_B$ for any basis B for V.

If $T:V\to V$ is a linear operator, we define $T^n=\overbrace{T\circ T\circ\cdots\circ T}^{n}$, and for a polynomial $f(t)=a_nt^n+a_{n-1}t^{n-1}+\cdots+a_1t+a_0$, define $f(T)=a_nT^n+a_{n-1}T^{n-1}+\cdots+a_1T+a_0\mathcal{I}$, where $\mathcal{I}$ is the identity map on V.

The observations above imply $[f(T)]_B=f([T]_B)$. Hence if $\Delta_T(t)$ is the characteristic polynomial of T, then
$$[\Delta_T(T)]_B=\left[\Delta_{[T]_B}(T)\right]_B=\Delta_{[T]_B}([T]_B)=0$$
by the Cayley-Hamilton Theorem. Thus the theorem holds for linear operators as well: $\Delta_T(T)$ is the zero operator.