(22) Lec 25, 3/30/09

## Eigenvalues and Eigenvectors (§9.4, §9.5)

**Defn:** Let $T: V \to V$ be a linear operator. A number $\lambda \in K$ is an eigenvalue of $T$ if there is a nonzero vector $\bar{v} \in V$ such that $T(\bar{v}) = \lambda \bar{v}$.
A **NONZERO** vector $\bar{v}$ satisfying $T(\bar{v}) = \lambda \bar{v}$ is called an eigenvector of $T$ belonging to $\lambda$.

Remarks:

① If $A$ is an $n \times n$ matrix, an eigenvalue/eigenvector of $A$ is defined to be an eigenvalue/eigenvector of $\varphi_A: K^n \to K^n$, $\varphi_A(\bar{x}) = A\bar{x}$. Thus $\lambda \in K$ is an eigenvalue of $A$ if $A\bar{v} = \lambda \bar{v}$ for some nonzero $\bar{v} \in V$, and such a vector $\bar{v}$ is an eigenvector of $A$.

② The zero vector is NEVER an eigenvector. Eigenvectors are **NONZERO** vectors. (The text is not clear on this point.)
However, $\lambda = 0$ can be an eigenvalue.

**Defn:** If $\lambda$ is an eigenvalue of $T$, the eigenspace of $T$ belonging to $\lambda$ is $E_\lambda = \{\bar{v} \in V \mid T(\bar{v}) = \lambda \bar{v}\}$.
(The eigenspace of a matrix $A$ is $E_\lambda = \{\bar{v} \in V \mid A\bar{v} = \lambda \bar{v}\}$.)

**Theorem:** If $\lambda$ is an eigenvalue of the linear operator $T$ on $V$, then $E_\lambda$ is a NONZERO subspace of $V$.

Proof: Exercise. [See HW #2, Problem 5 for the matrix version] □

Notes ① $E_\lambda$ consists of all eigenvectors of $T$ belonging to $\lambda$ along with the zero vector ($T(\bar{0}) = \bar{0} = \lambda \bar{0}$).