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(EXAMPLES, cont.)

2. $A=\begin{bmatrix}1&-2&1\\0&1&4\\0&0&2\end{bmatrix}$, $\Delta_A(t)=\begin{vmatrix}t-1&2&-1\\0&t-1&-4\\0&0&t-2\end{vmatrix}=(t-1)^2(t-2)$.

[Note: The eigenvalues of a triangular matrix are just the diagonal entries.]

Eigenspace for $\lambda=1$:

$E_1=\mathcal{N}(I-A)$, $I-A=\begin{bmatrix}0&2&-1\\0&0&-4\\0&0&-1\end{bmatrix}\longrightarrow\begin{bmatrix}0&2&-1\\0&0&1\\0&0&0\end{bmatrix}$.

Hence $E_1$ is the set of $\begin{bmatrix}x\\y\\z\end{bmatrix}$ such that $\begin{matrix}2y-z=0\\z=0\end{matrix}$ ($x$ is free),

hence $z=0$, $y=0$, and $x$ is free. Therefore

$E_1=\left\{\begin{bmatrix}x\\0\\0\end{bmatrix}\middle|\ x\in\mathbb{R}\right\}$, and $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}\right\}$ is a basis, $\underline{\dim E_1=1}$.

Eigenspace for $\lambda=2$:

$E_2=\mathcal{N}(2I-A)$, $2I-A=\begin{bmatrix}1&2&-1\\0&1&-4\\0&0&0\end{bmatrix}$; so $z$ is free, and

$y=4z$, $x=-2y+z=-8z+z=-7z$. Therefore,

$E_2=\left\{\begin{bmatrix}-7z\\4z\\z\end{bmatrix}\middle|\ z\in\mathbb{R}\right\}$, and $\left\{\begin{bmatrix}-7\\4\\1\end{bmatrix}\right\}$ is a basis, $\underline{\dim E_2=1}$

Note: Example 1: $\Delta_A(t)=(t-3)^2(t+3)$.

$\lambda=3$ is a root of multiplicity 2, $\dim E_3=2$,

$\lambda=-3$ is a root of multiplicity 1, $\dim E_{-3}=1$.

Example 2: $\Delta_A(t)=(t-1)^2(t-2)$.

$\lambda=1$ is a root of multiplicity 2, $\dim E_1=1$,

$\lambda=2$ is a root of multiplicity 1, $\dim E_2=1$.