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Diagonalizability

Defn: A square matrix A is <u>diagonalizable</u> if there is an invertible matrix P such that $D = P^{-1}AP$ is a diagonal matrix.

A linear operator $T: V \to V$ is <u>diagonalizable</u> if there is a basis B for V such that the matrix, $[T]_B$, of T relative to B is a diagonal matrix.

Remarks:

① If A is an $n \times n$ matrix and $\varphi = \varphi_A : K^n \to K^n$ is defined by $\varphi(\bar{x}) = A\bar{x}$, then $A = [\varphi]_E$, E the standard basis. If $D = P^{-1}AP = P^{-1}[\varphi]_E P$, then $D = [\varphi]_B$ relative to a basis B for K^n.

Hence A is a diagonalizable matrix if and only if $\varphi = \varphi_A$ is a diagonalizable linear operator.

② Suppose $T: V \to V$ is diagonalizable, so $[T]_B$ is a diagonal matrix for some basis B for V.

If S is any basis for V and P is the basis change matrix from S to B, then $P^{-1}[T]_S P = [T]_B$ is diagonal. Hence T is diagonalizable if and only if $[T]_S$ is diagonal<u>izable</u> for every basis S for V.

③ If $B = \{\bar{v}_1, \dots, \bar{v}_n\}$ is a basis for V such that $[T]_B = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$ is diagonal, then since the jth column of $[T]_B$ is $[T(\bar{v}_j)]_B$, we have $T(\bar{v}_j) = \lambda_j \bar{v}_j$. Thus the basis B consists of <u>eigenvectors</u> and the diagonal entries of $[T]_B$ are the corresponding eigenvalues.

The next result says that ③ holds for matrices as well.