<u>Corollary</u>: Let $T: V \to V$ be a linear operator with $\dim V = n$, A an $n \times n$ matrix. If T (or A) has n <u>distinct</u> eigenvalues, then T (or A) is diagonalizable.

<u>Corollary</u>: Let $\lambda_1, \cdots, \lambda_r$ be all of the <u>distinct</u> eigenvalues of $T: V \to V$ (or A). If B_i is a basis for the eigenspace E_{λ_i} for each i, then $B_1 \cup B_2 \cup \cdots \cup B_r$ is linearly independent.

<u>Proof</u>: Exercise. $\square$

<u>Remark</u>: The set $B_1 \cup B_2 \cup \cdots \cup B_r$ in the corollary is the largest possible linearly independent subset of V consisting of eigenvectors of T (or A).

<u>Corollary</u> Let $T: V \to V$ be a linear operator with $\dim V = n$, and assume

$$\Delta_T(t) = (t-\lambda_1)^{r_1}(t-\lambda_2)^{r_2} \cdots (t-\lambda_m)^{r_m}$$

with the $\lambda_i \in K$ <u>distinct</u> roots. The following are equivalent:

(i) T is diagonalizable.

(ii) $\dim E_{\lambda_1} + \dim E_{\lambda_2} + \cdots + \dim E_{\lambda_m} = n$.

(iii) $\dim E_{\lambda_i} = r_i$ for each i (that is, the algebraic multiplicity equals the geometric multiplicity of each eigenvalue).

<u>Proof</u>: (i)$\Rightarrow$(ii) If T is diagonalizable, there is a basis B of n eigenvectors. The vectors in B belonging to λ_i form a basis for E_{λ_i}; hence the sum of dimensions of the E_{λ_i} is $|B| = n = \dim V$.

(ii)$\Rightarrow$(iii) Suppose $\dim E_{\lambda_1} + \cdots + \dim E_{\lambda_m} = n$. We also have $r_1 + \cdots + r_m = n = \deg \Delta_T$. We showed before that $1 \le \dim E_{\lambda_i} \le r_i$ for each i, so $\dim E_{\lambda_1} + \cdots + \dim E_{\lambda_m} = r_1 + \cdots + r_m$ implies $\dim E_{\lambda_i} = r_i$, all i.

(iii)$\Rightarrow$(i) Suppose $\dim E_{\lambda_i} = r_i$ for all i, and let B_i be a basis for E_{λ_i}, so that $|B_i| = r_i$. By the corollary above, $B_1 \cup B_2 \cup \cdots \cup B_m$ is a linearly independent set of $r_1 + \cdots + r_m = n$ vectors, so is a basis for V consisting of eigenvectors. Hence T is diagonalizable. $\square$