We now have the following algorithm to determine if a matrix A is diagonalizable and, if so, to find P so that $D = P^{-1}AP$ is diagonal:

(1) Find $\Delta_A(t)$ and all distinct eigenvalues.

(2) Find a basis for each eigenspace.
  (a) If $\dim E_\lambda$ equals the multiplicity of λ as a root of $\Delta_A(t)$ for every eigenvalue λ, then A is diagonalizable.
  (b) If $\dim E_\lambda$ is less than the multiplicity of λ as a root of $\Delta_A(t)$ for one or more eigenvalues λ, then A is not diagonalizable.

(3) If condition 2(a) holds, then the union of the bases for the eigenspaces forms a basis $B = \{\bar{v}_1, \ldots, \bar{v}_n\}$ for K^n. If $P = [\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_n]$ is the matrix with $\bar{v}_j$ as jth column, then

$$P^{-1}AP = D = \begin{bmatrix} \lambda_1 & & & \bigcirc \\ & \lambda_2 & & \\ & & \ddots & \\ \bigcirc & & & \lambda_n \end{bmatrix},$$

where $\bar{v}_j$ is an eigenvector belonging to λ_j for each j.

Remark: The diagonal entries of D are eigenvalues of A, hence roots of $\Delta_A(t)$. Thus if $\Delta_A(t)$ does not factor over K as a product of linear factors, i.e., not all roots of A are in K, then A is not diagonalizable over K. It may be over a larger field.

For example, if $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ then $\Delta_A(t) = t^2+1$, which has no real roots. Hence A is not diagonalizable over $\mathbb{R}$. But over $\mathbb{C}$, $\Delta_A(t) = (t+i)(t-i)$ and if $P = \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$, then $P^{-1}AP = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$.