(EXAMPLES, Cont.)

4) What are the possible characteristic polynomials for a 5×5 matrix with minimal polynomial $m(t)=(t-6)^2(t-5)$?

We know $\deg \Delta_A(t)=5$ and $\Delta_A(t)=(t-6)^k(t-5)^l$, where $k+l=5$ and $k\geq 2$, $l\geq 1$. Hence we have

$\Delta_A(t)=(t-6)^2(t-5)^3$, or

$\Delta_A(t)=(t-6)^3(t-5)^2$, or

$\Delta_A(t)=(t-6)^4(t-5)$.

5) Find $m_A(t)$ for $A=\begin{bmatrix}1&2&-2\\-2&5&-2\\-6&6&-3\end{bmatrix}$.

We saw before that $\Delta_A(t)=(t-3)^2(t+3)$. Hence $m_A(t)=(t-3)(t+3)$ or $m_A(t)=(t-3)^2(t+3)$.

If $f(t)=(t-3)(t+3)$, then $f(A)=(A-3I)(A+3I)=$

$$\begin{bmatrix}-2&2&-2\\-2&2&-2\\-6&6&-6\end{bmatrix}\begin{bmatrix}4&2&-2\\-2&8&-2\\-6&6&0\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix},$$ so $f(A)=0$.

Thus $\boxed{m_A(t)=(t-3)(t+3)}$

In general, given $\Delta_A(t)$, we can write down all possibilities for $m_A(t)$. The one of <u>lowest degree</u> with A as a zero is then $m_A(t)$.

6) Find $m_A(t)$ for $A=\begin{bmatrix}1&-2&1\\0&1&4\\0&0&2\end{bmatrix}$

We saw before that $\Delta_A(t)=(t-1)^2(t-2)$, hence either $m_A(t)=(t-1)(t-2)$ or $m_A(t)=(t-1)^2(t-2)$. Since

$$(A-I)(A-2I)=\begin{bmatrix}0&-2&1\\0&0&4\\0&0&1\end{bmatrix}\begin{bmatrix}-1&-2&1\\0&-1&4\\0&0&0\end{bmatrix}=\begin{bmatrix}0&2&-8\\0&0&0\\0&0&0\end{bmatrix}\neq 0,$$ we know

$m_A(t)\neq(t-1)(t-2)$, hence $\boxed{m_A(t)=(t-1)^2(t-2)}$.