(EXAMPLES, cont.)

7. Let $A = \left[\begin{array}{ccc|cc} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ \hline 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{array}\right]$. Since A is upper triangular, $\Delta_A(t) = (t-2)^5$, and so $m_A(t) = (t-2)^k$, some k with $1 \le k \le 5$.

$$(A-2I) = \left[\begin{array}{ccc|cc} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \neq 0, \quad (A-2I)^2 = \left[\begin{array}{ccc|cc} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \neq 0,$$

but $(A-2I)^3 = 0$ (check). Hence $\boxed{m_A(t) = (t-2)^3.}$

Remarks: 1. An $n \times n$ matrix of the form $J = \begin{bmatrix} \lambda & 1 & 0 & \cdots & 0 \\ 0 & \lambda & 1 & \ddots & \vdots \\ \vdots & & \lambda & \ddots & 0 \\ & & & \ddots & 1 \\ 0 & \cdots & & 0 & \lambda \end{bmatrix}$

(that is, λ on diagonal, 1 on super diagonal), is a Jordan block of size n, and $m_J(t) = \Delta_J(t) = (t-\lambda)^n$.

2. If A is a block diagonal matrix, $A = \begin{bmatrix} A_1 & & & 0 \\ & A_2 & & \\ & & \ddots & \\ 0 & & & A_k \end{bmatrix}$,

then $\Delta_A(t) = \Delta_{A_1}(t)\Delta_{A_2}(t)\cdots\Delta_{A_k}(t)$ but

$m_A(t)$ is the LEAST COMMON MULTPLE of $m_{A_1}(t), m_{A_2}(t), \cdots, m_{A_k}(t)$.
[See Theorem 9.19]

Thus $m_A(t)$ is geneally not the product of the $m_{A_i}(t)$ [See Example 7]

3. If A is a block triangular matrix, $A = \begin{bmatrix} A_1 & & & * \\ & A_2 & & \\ & & \ddots & \\ 0 & & & A_k \end{bmatrix}$,

then we also have $\Delta_A(t) = \Delta_{A_1}(t)\Delta_{A_2}(t)\cdots\Delta_{A_k}(t)$.

However, in this case nothing can be said in general about $m_A(t)$ relative to the $m_{A_i}(t)$, beyond general results above.