Remarks:

1. The conditions in the theorem are slightly easier to verify in general than those in Theorem 4.2. We just need to show some vector $\bar{w}$ is in W, instead of specifically showing $\bar{0} \in W$.

2. It follows from the proof of the theorem that if W is a subspace of V, then W has the same zero vector and additive inverses as in V.

3. By [A2] and remark 2, every subspace must contain the zero vector of V. Hence if S is a subset of V and $\bar{0}_V$ is not in S, then S is not a subspace of V.

EXAMPLES:

1. $\{\bar{0}\}$ and V are subspaces of V for any V.
   We call $\{\bar{0}\}$ the trivial subspace (or zero subspace).
   Any subspace other than $\{\bar{0}\}$ is a non-trivial subspace.
   Any subspace other than V is a proper subspace.

2. $V = \mathbb{R}^3$, $W = \{(a,b,c) \mid b = a-c\}$ is a subspace.

(i) Since $0 = 0-0$, the vector $(0,0,0)$ is in W and $W \neq \emptyset$.

(ii) Let $\bar{w}_1 = (a,b,c)$ and $\bar{w}_2 = (a',b',c')$ be in W, so that $b = a-c$ and $b' = a'-c'$. Then
$$\bar{w}_1 + \bar{w}_2 = (a+a', b+b', c+c')$$
and $(a+a') - (c+c') = (a-c) + (a'-c') = b+b'$. Hence $\bar{w}_1 + \bar{w}_2 \in W$.

(iii) Let $\alpha \in \mathbb{R}$ and $\bar{w} = (a,b,c) \in W$, so that $b = a-c$.
Then $\alpha\bar{w} = \alpha(a,b,c) = (\alpha a, \alpha b, \alpha c)$,
and $\alpha a - \alpha c = \alpha(a-c) = \alpha b$. Hence $\alpha\bar{w} \in W$.

$\boxed{\text{End Lec 3}}$ Since (i), (ii), and (iii) hold, W is a subspace of V. ∎