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Matrix of T relative to B

Denote the vectors in B by $\bar{v}_1, \bar{v}_2, \bar{v}_3, \bar{v}_4, \bar{v}_5$, respectively.

$$T(\bar{v}_1) = A\bar{v}_1 = \begin{bmatrix}0\\-2\\0\\0\\1\end{bmatrix} = \bar{v}_1 \quad (\text{recall } \bar{v}_1 \in E_1).$$

$$T(\bar{v}_2) = A\bar{v}_2 = \begin{bmatrix}1\\2\\0\\0\\-1\end{bmatrix} = -\begin{bmatrix}0\\-2\\0\\0\\1\end{bmatrix} + \begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix} = -\bar{v}_1 + \bar{v}_2.$$

(observe that $T(\bar{v}_2) = 1\bar{v}_2 + \bar{\omega}$, where $\bar{\omega} \in E_1$, as expected)

$$T(\bar{v}_3) = A\bar{v}_3 = \begin{bmatrix}-1\\0\\-1\\1\\0\end{bmatrix} = -\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix} + \begin{bmatrix}0\\0\\-1\\1\\0\end{bmatrix} = -\bar{v}_2 + \bar{v}_3 = 0\bar{v}_1 - \bar{v}_2 + \bar{v}_3$$

(observe that $T(\bar{v}_3) = 1\bar{v}_3 + \bar{\omega}$, where $\bar{\omega} \in \ker(T - 1I)^2$, as expected)

$$T(\bar{v}_4) = \begin{bmatrix}0\\0\\-2\\4\\0\end{bmatrix} = 2\begin{bmatrix}0\\0\\-1\\2\\0\end{bmatrix} = 2\bar{v}_4 \quad (\text{recall } \bar{v}_4 \in E_2).$$

$$T(\bar{v}_5) = \begin{bmatrix}0\\-6\\0\\0\\4\end{bmatrix} = 2\begin{bmatrix}0\\-3\\0\\0\\2\end{bmatrix} = 2\bar{v}_5 \quad (\text{recall also } \bar{v}_5 \in E_2).$$

Hence
$$[T]_B = \left[\begin{array}{ccc|cc} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{array}\right]$$

[End Lec 32]

As expected, $[T]_B$ is triangular with the eigenvalues on the diagonal. If $P = [\bar{v}_1, \bar{v}_2, \bar{v}_3, \bar{v}_4, \bar{v}_5]$, then $P^{-1}AP = [T]_B$.