Recall that if $T: V \to V$ is a linear operator and W is a T-invariant subspace, then a basis S_0 for W can be extended to a basis S for V, and then

$$[T]_S = \left[\begin{array}{c|c} A & B \\ \hline O & C \end{array}\right], \text{ where } A = [\hat{T}]_{S_0}, \ \hat{T} \text{ the restriction of } T \text{ to } W.$$

<u>Theorem</u>: If T is a linear operator on V, W a T-invariant subspace, then $\Delta_{\hat{T}}(t)$ is a factor of $\Delta_T(t)$.

<u>Proof</u>: In the notation above, $\Delta_{\hat{T}}(t) = \Delta_A(t)$, and since $[T]_S$ is block triangular, $\Delta_T(t) = \Delta_A(t)\Delta_C(t) = \Delta_{\hat{T}}(t)\Delta_C(t)$. $\square$

<u>Remark</u>: If W is T-invariant and $f(t)$ is any polynomial, then W is $f(T)$-invariant [Exercise.]
Also, $\widehat{f(T)} = f(\hat{T})$. [See Problem 10.6(a).]

<u>Theorem</u>: If T is a linear operator on V, W a T-invariant subspace, then $m_{\hat{T}}(t)$ is a factor of $m_T(t)$.

<u>Proof</u>: We know $m_T(T)$ is the zero map on V, hence $\widehat{m_T(T)}$ is the zero map on W. By the remark above $m_T(\hat{T})$ is the zero map on W. Hence $m_T(t)$ is a polynomial with $\hat{T}$ as a zero, and so by a previous theorem, $m_{\hat{T}}(t)$ is a factor of $m_T(t)$. $\square$

<u>Note</u>: In general, $m_{\hat{T}}(t) = m_A(t)$, but $m_T(t) \neq m_A(t)\,m_C(t)$, in the notation above.