AVERAGING FOR SYSTEMS OF STOCHASTIC EQUATIONS WITH RANDOM DISTURBANCES

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Abstract. We consider the systems of stochastic differential equations. The coefficients of the equations depend on a small parameter. The first equation, "slow" component, has unbounded drift and random disturbances, which are described by the second equation, "fast" component, with periodic coefficients. The sufficient conditions for weak convergence of the solutions of the "slow" equations to the certain random process are proved.

1.Introduction

In the paper we consider the systems of stochastic equations with small parameter ε. The first equation, "slow" component, has unbounded drift and coefficients which depend on the small parameter. We can consider the second equations as random disturbances of the first equation. These random disturbances, "fast" component, are described by the Markov diffusion processes with periodic coefficients. We will study the weak convergence of probability measures induced by the "slow" equations to a certain random process.

An asymptotic behavior of the undisturbed solutions of the stochastic equations with unbounded drift seems to be considered for the first time in the papers of G. Kulinuch [5] and of N. Portenko [8].

In the case, when the coefficients of the first equations do not depend on random disturbances, sufficient conditions (necessary and sufficient conditions) of the weak convergence of solutions in more general situations are obtained by S. Makhno [6] ([7]).

In the case, when the coefficients of the first equations do not depend on the small parameter, and unbounded drift is absent, the problems of the weak convergence of solutions under the various conditions on the coefficients and random disturbances have been studying by many authors (see, e.g. monograph of A. Skorokhod [10], and bibliography).

Our aim is to join the results of these two directions in simple situation. In the second part of the proof of the Theorem we use extensively the technique from S. Makhno [7].

The organization of the paper is as follows. In this section we set up some notations and assumptions. In the next section we formulate the main result. In the section 3 we study the preliminary results and in section 4 we consider the example.

Let (Ω, F, P) denote some probability space with filtration F_t, $t \in [0, T]$. Let E_n be a n-dimensional Euclidean space, $E_+ = [0, +\infty)$, symbol $\mathbf{E}$ denotes the mathematical expectation, $\dot{f}(x)$ be a derivative of the function $f(x)$, and ∇_y is the symbol of the gradient with respect to $y \in E_n$. We denote different positive constant by C with indexes if need.

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Let us consider a system of stochastic equations, $t \in [0,T]$,

$$\xi_\varepsilon(t) = \xi_0 + \int_0^t \frac{1}{\varepsilon^\delta} b\Big(\frac{\xi_\varepsilon(s)}{\varepsilon^\delta}, \eta_\varepsilon(s)\Big) ds + \int_0^t g\Big(\frac{\xi_\varepsilon(s)}{\varepsilon^\delta}, \eta_\varepsilon(s)\Big) ds + \int_0^t \sigma\Big(\frac{\xi_\varepsilon(s)}{\varepsilon^\delta}, \eta_\varepsilon(s)\Big) dw(s), \tag{1.1}$$

$$\eta_\varepsilon(t) = \eta_0 + \frac{1}{\varepsilon}\int_0^t g_1\left(\eta_\varepsilon(s)\right) ds + \frac{1}{\sqrt{\varepsilon}}\int_0^t \sigma_1\left(\eta_\varepsilon(s)\right) dw_1(s). \tag{1.2}$$

Here $\{w(t), F_t\}$ is one-dimensional standard Wiener process, $\{w_1(t), F_t\}$ is n-dimensional standard Wiener process. The processes $w(t)$ and $w_1(t)$ are independent. The processes $\xi_\varepsilon(t) \in E_1$,$\eta_\varepsilon(s) \in E_n$; the constants ξ_0, η_0 are non random; $b(x,y)$,$g(x,y)$, and $\sigma(x,y)$ are the functions from $E_1 \times E_n$ in E_1;$g_1(y)$, $\sigma_1(y)$ are the functions from E_n in E_n and $\mathcal{L}(E_n)$ respectively; $\varepsilon > 0$ is a small parameter; δ is a fixed number from $]0, \frac{1}{3}[$. If the equation (1.2) has a unique (in sense of law) weak solution, then the distribution $\eta_\varepsilon(t\varepsilon)$ coincides with the distribution of process $\eta(t)$ – the solution of the Ito stochastic equation

$$d\eta(t) \;=\; g_1(\eta(t))dt + \sigma_1(\eta(t))dw_1(t)$$

and does not depend from ε.

Let us denote by $C^{k,l}_{x,y}(E_1, E_n)$ the class of the functions $f(x,y)$ k, and l times continuously diffferentiable with respect to $x \in E_1$, and $y \in E_n$ respectively; the symbol "b" in the notation of this class ($C^{k,l}_{x,y,b}(E_1, E_n)$) indicates that these functions and their derivatives of the stipulated order with respect to $x \in E_1$ are bounded. Let $a^1_{ij}(y)$ be the components of the $n \times n$ matrix $a^1(y) = \sigma_1(y)\sigma'_1(y)$, and $g^1_i(y)$ be the components of the vector $g_1(y)$.

We want to introduce condition (A).

Condition (A).

A1. The functions $a^1_{ij}(y)$, $g^1_i(y) \in C^2_y(E_n)$ and are periodic of period 1 in y ;

A2. There exists a constant $\lambda_0 > 0$ such that for every $y, \zeta \in E_n$

$$a^1_{ij}(y)\zeta_i\zeta_j \geq \lambda_0|\zeta|^2.$$

Denote by L^* the operator which is formally conjugate to the generating operator L of η_t

$$L \;=\; \frac{1}{2}\sum_{i,j=1}^n a^1_{ij}(y)\frac{\partial^2}{\partial y_i \partial y_2} + \sum_{i=1}^n g^1_i(y)\frac{\partial}{\partial y_i}.$$

We shall denote by Y the unit torus in E_n. As well known (see, for example, [1]) the next problem

$$L^*p(y) \;=\; 0, \qquad \int_Y p(y)dy \;=\; 1$$

has the unique positive periodic of period 1 solution $p(y)$ and for a periodic of period 1 function $h(y)$ such that

$$\int_Y h(y)p(y)dy \;=\; 0$$

the next problem

$$Ld(y) \;=\; h(y), \qquad \int_Y d(y)dy \;=\; 0$$

has the unique periodic of period 1 solutions $d(y) \in C_y^2(E_n)$.

We will need the estimations of the solution of the next problem

$$Ld(x,y) \;=\; h(x,y), \qquad \int_Y d(x,y)dy \;=\; 0 \tag{1.3}$$

and its first two derivatives with respect to the parameter x. We give the proof of such result belonging to M. Safonov [9]: (Below we denote by ∂Y the boundary of Y)

Lemma 2.1. *Let $d = d\,(y)$ be a periodic function satisfying*

$$Ld\,(y) = h\,(y)\,, \qquad \int_Y d\,(y)\,dy = 0.$$

Then

$$\sup_Y |d| \le C \sup_Y |h|\,,$$

with a constant C depending only on the prescribed quantities, such as dimension n, ellipticity constant λ_0, etc.

Proof. Let Y_2 denote a cube concentric with Y, and of twice bigger size than Y. We can represent d in the form $d = d_0 + d_1$, where

$$Ld_0 = 0 \quad \text{in} \quad Y_2, \quad d_0 = d \quad \text{on} \quad \partial Y_2;$$

$$Ld_1 = h \quad \text{in} \quad Y_2, \quad d_1 = 0 \quad \text{on} \quad \partial Y_2.$$

By the interior Harnack inequality, the classical maximum principle (see [2]), and periodicity of d,

$$\operatorname{osc}\{d_0;\; Y\} = \sup_Y d_0 - \inf_Y d_0 \le \theta \operatorname{osc}\{d_0;\; Y_2\} = \theta \operatorname{osc}\{d;\; Y_2\} = \theta \operatorname{osc}\{d;\; Y\}\,,$$

with $\theta = \theta\,(n, \lambda, \dots) < 1$. Moreover, by an elementary estimate (which follows from the maximum principle),

$$\sup |d_1| \le C_1 \sup |h|\,.$$

These estimates imply

$$\operatorname{osc}\{d;\; Y\} \le \operatorname{osc}\{d_0;\; Y\} + \operatorname{osc}\{d_1;\; Y\} \le \theta \operatorname{osc}\{d;\; Y\} + 2C_1 \sup |h|\,,$$

$$\operatorname{osc}\{d;\; Y\} \le \frac{2C_1}{1-\theta} \sup |h|\,,$$

and the desired estimate follows. □

Corollary. *Under the previous assumptions, let d and h depend on a parameter x. Then the derivatives of d and h with respect to x satisfy*

$$\sup_Y \left|\frac{\partial d}{\partial x}\right| \leq C \sup_Y \left|\frac{\partial h}{\partial x}\right|.$$

Proof. Proof follows immediately from linearity. □

Let

$$\bar{d}(x) = \langle d(x,\ \cdot\ )\rangle \triangleq \int_Y d(x,y)p(y)dy.$$

Thus, if $\bar{h}(x) = 0$ and $h(x,y)$ belonging to the class $C^{2,2}_{x,y,b}(E_1, E_n)$ is periodic of period 1 with respect to $y \in E_n$, then the solution of (1.3) $d(x,y) \in C^{2,2}_{x,y,b}(E_1, E_n)$.

We set $h_\varepsilon(x,y) = h(\frac{x}{\varepsilon^\delta}, y)$, and $l_\varepsilon(x) = l(\frac{x}{\varepsilon^\delta})$. Setting $\psi_1(x,y) = h_\varepsilon(x,y)$ and $\psi_2(x) = l_\varepsilon(x)$, we note that $\frac{\partial}{\partial x}\psi_1(x,y) = \frac{1}{\varepsilon^\delta}\frac{\partial}{\partial x}h_\varepsilon(x,y)$, $\frac{\partial^2}{\partial x^2}\psi_1(x,y) = = \frac{1}{\varepsilon^{2\delta}}\frac{\partial^2}{\partial x^2}h_\varepsilon(x,y)$; $\dot{\psi}_2(x) = \frac{1}{\varepsilon^\delta}\dot{l}_\varepsilon(x)$, $\ddot{\psi}_2(x) = \frac{1}{\varepsilon^{2\delta}}\ddot{l}_\varepsilon(x)$. These symbols we use for the notation of the derivatives of a function, which depends on ε.

Let us denote $a(x,y) = \sigma^2(x,y)$ and introduce condition (B).

Condition (B).

B1. The functions $a(x,y)$, $g(x,y)$, $b(x,y) \in C^{2,2}_{x,y,b}(E_1, E_n)$ are periodic of period 1 in y;

B2. There exists the constant $\lambda_1 > 0$ such that for every $x \in E_1$

$$\overline{a}(x) \geq \lambda_1;$$

B3. There exists the constant $\lambda_2 > 0$ such that for every $x \in E_1$

$$\left|\int_0^x \frac{\overline{b}(z)}{\overline{a}(z)}dz\right| \leq \lambda_2.$$

Under the conditions (A), (B1) the system ((1,1), (1,2)) has the unique strong solution.

Denote

$$F(x) = \exp\left\{-2\int_0^x \frac{\overline{b}(z)}{\overline{a}(z)}dz\right\}, \qquad h(x) = \int_0^x F(z)dz.$$

Condition (C).

There exist next limits

$$\text{C0.} \lim_{|Z|\to\infty} \frac{1}{Z}\int_0^Z F(x)dx = \alpha;$$

$$\text{C1.} \lim_{|Z|\to\infty} \frac{1}{Z}\int_0^Z \frac{1}{\overline{a}(x)F(x)}dx = \alpha_1;$$

$$\text{C2.} \lim_{|Z|\to\infty} \frac{1}{Z}\int_0^Z \frac{\overline{g}(x)}{\overline{a}(x)}dx = \alpha_2;$$

Remark. It follows from Lemma 3.1 and our conditions that there exist constants C_1, C_2 such that

$$0 < C_1 < \alpha < C_2\,; \quad 0 < C_1 < \alpha_1 < C_2\,; \quad |\alpha_2| < C_2$$

Let

$$f(x) = \frac{1}{\alpha}h(x) - x.$$

Obviously

$$\overline{L}_x f(x) \triangleq \bar{b}(x)\dot{f}(x) + \frac{1}{2}\bar{a}(x)\ddot{f}(x) \;=\; -\bar{b}(x). \tag{1.4}$$

Let denote by $b^0(x,y)$ the function such that

$$L_x f(x) \triangleq b(x,y)\dot{f}(x) + \frac{1}{2}a(x,y)\ddot{f}(x) = b^0(x,y). \tag{1.5}$$

It is clear $\bar{b}^0(x) \;=\; -\bar{b}(x)$. Let

$$\tilde{L}_x^\varepsilon \;=\; \frac{1}{\varepsilon^\delta}b\left(\frac{x}{\varepsilon^\delta},y\right)\frac{d}{dx} + \frac{1}{2}a\left(\frac{x}{\varepsilon^\delta},y\right)\frac{d^2}{dx^2},$$

thus we have

$$\tilde{L}_x^\varepsilon f_\varepsilon(x) \;=\; \frac{1}{\varepsilon^{2\delta}}b\left(\frac{x}{\varepsilon^\delta},y\right)\dot{f}_\varepsilon(x) + \frac{1}{2\varepsilon^{2\delta}}a\left(\frac{x}{\varepsilon^\delta},y\right)\ddot{f}_\varepsilon(x) \triangleq \frac{1}{\varepsilon^{2\delta}}L_x^\varepsilon f_\varepsilon(x). \tag{1.6}$$

Let $(\mathcal{C}[0,T],\mathcal{C}_t)$, $t \in [0,T]$ be a space of all continuous functions on $[0,T]$, a space C_0^∞ is the space of all infinitely differentiable functions with compact support on E_1. Denote by $\{\mu_\delta^\varepsilon, \varepsilon > 0\}$ the family of probability measures induced by the random processes $\{\xi_\varepsilon(t), \varepsilon > 0\}$ on $\mathcal{C}([0,T])$ and by "$\Rightarrow$" the sign for weak convergence of measures. We will prove weak convergence

$$\mu_\delta^\varepsilon \Rightarrow \mu$$

as ε tends to 0 for each δ, where μ is the measure corresponding to the random process

$$\xi(t) \;=\; \xi_0 \;+\; \beta_0 t \;+\; \sigma_0 w(t) \tag{1.7}$$

with certain constant coefficients.

2.Main result

In this section we give the proof of the limit theorem about weak convergence of the solutions (1.1) to the process (1.7).

Theorem. *Let conditions $(A),(B),(C)$ be fulfilled. Then for every $\delta \in]0,\frac{1}{3}[$ the measures $\mu_\delta^\varepsilon \Rightarrow \mu$ as ε tends to 0.The random process $\xi(t)$, which corresponds to μ, is defined by (1.7), where*

$$\beta_0 \;=\; \frac{\alpha_2}{\alpha\alpha_1}, \quad \sigma_0 \;=\; \frac{1}{\sqrt{\alpha\alpha_1}}.$$

Proof.

First (I), we prove that for each $\delta \in]0,\frac{1}{3}[$ the family of measures $\{\mu_\delta^\varepsilon,\, \varepsilon > 0\}$, is weakly compact on $\mathcal{C}[0,T]$, and, second (II), we are going to the limit as ε tends to 0, giving the possibility to the coefficients of equations (1.1) to obtain the averaging form with respect to random perturbations and after that we use the condition (C) for identification of the limit process.

I. Now we fix arbitrary $\delta \in]0, \frac{1}{3}[$. By Ito formula we obtain

$$\varepsilon^{\delta}(f_{\varepsilon}(\xi_{\varepsilon}(t)) - f_{\varepsilon}(\xi_0)) = \varepsilon^{-\delta}\int_0^t L^{\varepsilon}_{\xi_{\varepsilon}(s)} f_{\varepsilon}(\xi_{\varepsilon}(s))ds + \int_0^t \dot{f}_{\varepsilon}(\xi_{\varepsilon}(s))g_{\varepsilon}(\xi_{\varepsilon}(s),\eta_{\varepsilon}(s))ds + \int_0^t \dot{f}_{\varepsilon}(\xi_{\varepsilon}(s))\sigma_{\varepsilon}(\xi_{\varepsilon}(s),\eta_{\varepsilon}(s))dw(s) = \varepsilon^{-\delta}\int_0^t b^0_{\varepsilon}(\xi_{\varepsilon}(s),\eta_{\varepsilon}(s))ds + \int_0^t \dot{f}_{\varepsilon}(\xi_{\varepsilon}(s))g_{\varepsilon}(\xi_{\varepsilon}(s),\eta_{\varepsilon}(s))ds + \int_0^t \dot{f}_{\varepsilon}(\xi_{\varepsilon}(s))\sigma_{\varepsilon}(\xi_{\varepsilon}(s),\eta_{\varepsilon}(s))dw(s). \tag{2.1}$$

Let us denote $B(x,y) = b(x,y) + b^0(x,y)$, where $b^0(x,y)$ was defined by (1.5).

Taking into account the equality $\dot{f}_{\varepsilon}(x) = \frac{1}{\alpha}F_{\varepsilon}(x) - 1$, the sum of (1.1) and (2.1) gives

$$\xi_{\varepsilon}(t) + \varepsilon^{\delta} f_{\varepsilon}(\xi_{\varepsilon}(t)) = \xi_0 + \varepsilon^{\delta} f_{\varepsilon}(\xi_0) + \varepsilon^{-\delta}\int_0^t B_{\varepsilon}(\xi_{\varepsilon}(s),\eta_{\varepsilon}(s))ds + \frac{1}{\alpha}\int_0^t F_{\varepsilon}(\xi_{\varepsilon}(s))g_{\varepsilon}(\xi_{\varepsilon}(s),\eta_{\varepsilon}(s))ds + \frac{1}{\alpha}\int_0^t F_{\varepsilon}(\xi_{\varepsilon}(s))\sigma_{\varepsilon}(\xi_{\varepsilon}(s),\eta_{\varepsilon}(s))dw(s). \tag{2.2}$$

Let the function $m(x,y)$ be the unique periodic of period 1 with respect to y solution of

$$Lm(x,y) = B(x,y), \qquad \int_Y m(x,y)dy = 0 \tag{2.3}$$

for every $x \in E_1$ (x - parameter), because $\bar{B}(x) = 0$.

Applying Ito formula to the function $\varepsilon^{1-\delta} m_{\varepsilon}(\xi_{\varepsilon}(t),\eta_{\varepsilon}(t))$, we get

$$\varepsilon^{1-\delta}(m_{\varepsilon}(\xi_{\varepsilon}(t),\eta_{\varepsilon}(t)) - m_{\varepsilon}(\xi_0,\eta_0)) = \int_0^t A_0(\varepsilon,s)ds + \int_0^t A_1(\varepsilon,s)ds + \int_0^t A_2(\varepsilon,s)dw(s) + \int_0^t A_3(\varepsilon,s)dw_1(s) + \varepsilon^{-\delta}\int_0^t Lm_{\varepsilon}(\xi_{\varepsilon}(s),\eta_{\varepsilon}(s))ds, \tag{2.4}$$

here

$$\begin{aligned} A_0(\varepsilon,s) &= \varepsilon^{1-3\delta} L^{\varepsilon}_{\xi_{\varepsilon}(s)} m_{\varepsilon}(\xi_{\varepsilon}(s),\eta_{\varepsilon}(s));\\ A_1(\varepsilon,s) &= \varepsilon^{1-2\delta} g_{\varepsilon}(\xi_{\varepsilon}(s),\eta_{\varepsilon}(s))\frac{\partial m_{\varepsilon}}{\partial x}(\xi_{\varepsilon}(s),\eta_{\varepsilon}(s));\\ A_2(\varepsilon,s) &= \varepsilon^{1-2\delta} \sigma_{\varepsilon}(\xi_{\varepsilon}(s),\eta_{\varepsilon}(s))\frac{\partial m_{\varepsilon}}{\partial x}(\xi_{\varepsilon}(s),\eta_{\varepsilon}(s));\\ A_3(\varepsilon,s) &= \varepsilon^{\frac{1}{2}-\delta}\sigma_1(\eta_{\varepsilon}(s))\nabla_y m_{\varepsilon}(\xi_{\varepsilon}(s),\eta_{\varepsilon}(s)). \end{aligned} \tag{2.5}$$

Note that for every $t \in [0,T]$ there exists the constant C such that

$$|A_0(\varepsilon,t)| \le \varepsilon^{1-3\delta}C;\ |A_1(\varepsilon,t)| + |A_2(\varepsilon,t)| \le \varepsilon^{1-2\delta}C;\ |A_3(\varepsilon,t)| \le \varepsilon^{\frac{1}{2}-\delta}C. \tag{2.6}$$

We denote

$$\zeta_{\varepsilon}(t) = \xi_{\varepsilon}(t) + \varepsilon^{\delta} f_{\varepsilon}(\xi_{\varepsilon}(t)) - \varepsilon^{1-\delta} m_{\varepsilon}(\xi_{\varepsilon}(t),\eta_{\varepsilon}(t)). \tag{2.7}$$

From the relationship (2.2), using (2.3) and (2.4), we get

$$\begin{aligned}\zeta_\varepsilon(t) = \zeta_\varepsilon(0) + \int_0^t \left[\frac{1}{\alpha}F_\varepsilon(\xi_\varepsilon(s))g_\varepsilon(\xi_\varepsilon(s),\eta_\varepsilon(s)) - A_0(\varepsilon,s) - A_1(\varepsilon,s)\right]ds+ \\ \int_0^t \left[\frac{1}{\alpha}F_\varepsilon(\xi_\varepsilon(s))\sigma_\varepsilon(\xi_\varepsilon(s),\eta_\varepsilon(s)) - A_2(\varepsilon,s)\right]dw(s) - \int_0^t A_3(\varepsilon,s)dw_1(s).\end{aligned} \tag{2.8}$$

By virtue Lemma 3.1 from (2.6) and the condition (B) we have

$$\begin{gathered}\left|\frac{1}{\alpha}F_\varepsilon(\xi_\varepsilon(s))g_\varepsilon(\xi_\varepsilon(s),\eta_\varepsilon(s)) - A_0(\varepsilon,s) - A_1(\varepsilon,s)\right| + \\ \left|\frac{1}{\alpha}F_\varepsilon(\xi_\varepsilon(s))\sigma_\varepsilon(\xi_\varepsilon(s),\eta_\varepsilon(s)) - A_2(\varepsilon,s)\right| + |A_3(\varepsilon,s)| \le C(1+C_\varepsilon),\end{gathered}$$

where $\lim_{\varepsilon\to 0} C_\varepsilon = 0$. From this and (2.8) by standard argument we obtain that for arbitrary fixed $\varepsilon_0 > 0$ exists a constant C_{ε_0} such that for every $0 < \varepsilon < \varepsilon_0$

$$\mathbf{E}\sup_{t\in[0,T]} |\zeta_\varepsilon(t)|^2 \le C_{\varepsilon_0}(1+|\xi_0|^2),$$

and for every $s,t:\quad 0 \le s \le t \le T$

$$\mathbf{E}|\zeta_\varepsilon(t) - \zeta_\varepsilon(s)|^4 \le C_{\varepsilon_0}|t-s|^2.$$

Using (2.7) and Lemma 3.4, we can check the conditions of weakly compactness [3, Lemma 2, p.355] for the family of measures $\{\mu_\delta^\varepsilon,\ 0<\varepsilon<\varepsilon_0\}$. Thus the set of measures corresponding to the processes

$$\xi_\varepsilon(t) = \zeta_\varepsilon(t) - \varepsilon^\delta f_\varepsilon(\xi_\varepsilon(t)) + \varepsilon^{1-\delta}m_\varepsilon(\xi_\varepsilon(t),\eta_\varepsilon(t))$$

on $\mathcal{C}[0,T]$ is weakly compact.

II. We begin our consideration with the relationship (2.8).

Let $\phi(x) \in C_0^\infty$, $\Phi_s(x)$ be a continuous bounded $\mathcal{C}_s$ — measurable functional. Applying Ito formula to (2.8), we obtain

$$\begin{gathered}\mathbf{E}\Phi_r(\xi_\varepsilon)\left[\phi(\zeta_\varepsilon(t)) - \phi(\zeta_\varepsilon(r))\right] = \\ \mathbf{E}\Phi_r(\xi_\varepsilon)\left\{\int_r^t \dot\phi(\zeta_\varepsilon)\left[\frac{1}{\alpha}F_\varepsilon(\xi_\varepsilon(s))g_\varepsilon(\xi_\varepsilon(s),\eta_\varepsilon(s)) + A_0(\varepsilon,s) + A_1(\varepsilon,s)\right]ds+ \right.\\ \left.\frac{1}{2}\int_r^t \ddot\phi(\zeta_\varepsilon)\left[\left(\frac{1}{\alpha}F_\varepsilon(\xi_\varepsilon(s))\sigma_\varepsilon(\xi_\varepsilon(s),\eta_\varepsilon(s)) + A_2(\varepsilon,s)\right)^2 + A_3^2(\varepsilon,s)\right]ds\right\}.\end{gathered} \tag{2.9}$$

Now, we denote

$$\begin{gathered}D_\varepsilon(r,t) = \int_r^t \left[\dot\phi(\zeta_\varepsilon)(A_0(\varepsilon,s) + A_1(\varepsilon,s)) + \right.\\ \left.\ddot\phi(\zeta_\varepsilon)\left(\frac{1}{\alpha}F_\varepsilon(\xi_\varepsilon(s))\sigma_\varepsilon(\xi_\varepsilon(s),\eta_\varepsilon(s))A_2(\varepsilon,s) + \frac{1}{2}A_2^2(\varepsilon,s) + \frac{1}{2}A_3^2(\varepsilon,s)\right)\right]ds.\end{gathered} \tag{2.10}$$

Applying Lemma 3.3 for $n(x,y) = g(x,y) - \langle g(x,\,\cdot\,)\rangle$, $H_\varepsilon(x) = F_\varepsilon(x)$, and $\psi(z) = \dot\phi(z)$, taking into account denotation (3.3), we arrive at

$$\begin{gathered}\mathbf{E}\Phi_r(\xi_\varepsilon)\left[\int_r^t \dot\phi(\zeta_\varepsilon(s))F_\varepsilon(\xi_\varepsilon(s))g_\varepsilon(\xi_\varepsilon(s),\eta_\varepsilon(s))ds\right] = \\ \mathbf{E}\Phi_s(\xi_\varepsilon)\left[\int_r^t \dot\phi(\zeta_\varepsilon(s))F_\varepsilon(\xi_\varepsilon(s))\overline{g}_\varepsilon(\xi_\varepsilon(s))ds + G_\varepsilon(g_\varepsilon - \overline{g}_\varepsilon, r, t)\right]\end{gathered} \tag{2.11}$$

By similar way, applying Lemma 3.3 for $n(x,y) = a(x,y) - \langle a(x,\,\cdot\,)\rangle$, $H_\varepsilon(x) = F_\varepsilon^2(x))$, and $\psi(z) = \ddot\phi(z)$ we can obtain

$$\begin{gathered}\mathbf{E}\Phi_r(\xi_\varepsilon)\left[\int_r^t \ddot\phi(\zeta_\varepsilon(s))F_\varepsilon^2(\xi_\varepsilon(s))a_\varepsilon(\xi_\varepsilon(s),\eta_\varepsilon(s))ds\right] = \\ \mathbf{E}\Phi_r(\xi_\varepsilon)\left[\int_r^t \ddot\phi(\zeta_\varepsilon(s))F_\varepsilon^2(\xi_\varepsilon(s))\overline{a}_\varepsilon(\xi_\varepsilon(s))ds + G_\varepsilon(a_\varepsilon - \overline{a}_\varepsilon, r, t)\right]\end{gathered} \tag{2.12}$$

Replacing the terms in (2.9) by the right hand sides of (2.11) and (2.12) and taking into account (2.10), we arrive at

$$\begin{gathered}\mathbf{E}\Phi_r(\xi_\varepsilon)\left[\phi(\zeta_\varepsilon(t)) - \phi(\zeta_\varepsilon(r)) - \int_r^t \dot\phi(\zeta_\varepsilon)\frac{1}{\alpha}F_\varepsilon(\xi_\varepsilon(s))\left\langle g_\varepsilon(\xi_\varepsilon(s),\,\cdot\,)\right\rangle ds - \right. \\ \left. \frac{1}{2}\int_0^t \ddot\phi(\zeta_\varepsilon)\alpha^{-2}F_\varepsilon^2(\xi_\varepsilon(s))\left\langle a_\varepsilon(\xi(s),\,\cdot\,)\right\rangle ds\right] = I_\varepsilon^0,\end{gathered} \tag{2.13}$$

where

$$I_\varepsilon^0 = \mathbf{E}\Phi_r(\xi_\varepsilon)\left[\frac{1}{\alpha}G_\varepsilon(g - \langle g\rangle, r, t) + \frac{1}{\alpha}G_\varepsilon(a - \langle a\rangle, r, t) + D_\varepsilon(r,t)\right]. \tag{2.14}$$

Rewriting (2.13), we arrive at

$$\begin{gathered}\mathbf{E}\Phi_r(\xi_\varepsilon)\left[\phi(\xi_\varepsilon(t)) - \phi(\xi_\varepsilon(r)) - \int_r^t \{\dot\phi(\xi_\varepsilon(s))\beta_0 + \frac{1}{2}\ddot\phi(\xi_\varepsilon(s))\sigma_0^2\}ds\right] = \\ I_\varepsilon^0 + I_\varepsilon^1 + I_\varepsilon^2 + I_\varepsilon^3 + I_\varepsilon^4,\end{gathered} \tag{2.15}$$

here I_ε^0 defined by (2.14) and

$$I_\varepsilon^1 = \mathbf{E}\Phi_r(\xi_\varepsilon)\Big[\phi(\xi_\varepsilon(t)) - \phi(\zeta_\varepsilon(t)) - \phi(\xi_\varepsilon(r)) + \phi(\zeta_\varepsilon(r))\Big],$$

$$\begin{gathered}I_\varepsilon^2 = \mathbf{E}\Phi_r(\xi_\varepsilon)\int_r^t \left[\Big(\dot\phi(\zeta_\varepsilon(s)) - \dot\phi(\xi_\varepsilon(s))\Big)\frac{1}{\alpha}F_\varepsilon(\xi_\varepsilon(s))\left\langle g_\varepsilon(\xi_\varepsilon(s),\,\cdot\,)\right\rangle + \right. \\ \left. \frac{1}{2}\Big(\ddot\phi(\zeta_\varepsilon(s)) - \ddot\phi(\xi_\varepsilon(s))\Big)\alpha^{-2}F_\varepsilon^2(\xi_\varepsilon(s))\left\langle a_\varepsilon(\xi(s),\,\cdot\,)\right\rangle\right] ds,\end{gathered}$$

$$I_\varepsilon^3 = \mathbf{E}\Phi_r(\xi_\varepsilon)\int_r^t \dot\phi(\xi_\varepsilon(s))\left[\frac{1}{\alpha}F_\varepsilon(\xi_\varepsilon(s))\langle g_\varepsilon(\xi_\varepsilon(s),\,\cdot\,)\rangle - \beta_0\right]ds,$$

$$I_\varepsilon^4 = \frac{1}{2}\mathbf{E}\Phi_r(\xi_\varepsilon)\int_r^t \ddot\phi(\xi_\varepsilon(s))\Big[\alpha^{-2}F_\varepsilon^2(\xi_\varepsilon(s))\langle a_\varepsilon(\xi_\varepsilon(s),\,\cdot\,)\rangle - \sigma_0^2)\Big]ds.$$

We shall prove that the limit of the right hand side (2.15) is equal to zero. For small ε we can estimate $D_\varepsilon(r,t)$ (see (2.10)), using (2.6), condition (B) and Lemma 3.1,

$$\mathbf{E}\sup_{t\in[0,T]}|D_\varepsilon(t,r)|\le\varepsilon^{1-3\delta}C_T.$$

From this inequality and Lemma 3.3 (see (3.3)) we have

$$\lim_{\varepsilon\to0}I^0_\varepsilon=0. \tag{2.16}$$

According to the condition (B), Lemma 3.1 and , Lemma 3.4 we get

$$\lim_{\varepsilon\to0}I^1_\varepsilon=\lim_{\varepsilon\to0}I^2_\varepsilon=0. \tag{2.17}$$

We want to prove that

$$\lim_{\varepsilon\to0}I^3_\varepsilon=0. \tag{2.18}$$

According to Lemma 3.5, sufficiently to show that for every $r,t: 0\le r<t\le T$

$$\lim_{\varepsilon\to0}\mathbf{E}\left|\int_r^t\left[\frac{1}{\alpha}F_\varepsilon(\xi_\varepsilon(s))\langle g_\varepsilon(\xi_\varepsilon(s),\ \cdot\ )\rangle-\beta_0\right]ds\right|=0. \tag{2.19}$$

Let us denote

$$\gamma(x)=2\int_0^x F(z)\int_0^z\frac{\frac{1}{\alpha}F(y)\overline{g}(y)-\beta_0}{F(y)\overline{a}(y)}dydz. \tag{2.20}$$

Applying Ito formula to the function $\varepsilon^{2\delta}\gamma_\varepsilon(\xi_\varepsilon(t))$, we get

$$\begin{gathered}\varepsilon^{2\delta}\left(\gamma_\varepsilon(\xi_\varepsilon(t))-\gamma_\varepsilon(\xi_\varepsilon(t))\right)=\int_r^t\{b_\varepsilon(\xi_\varepsilon(s),\eta_\varepsilon(s))\dot{\gamma}_\varepsilon(\xi_\varepsilon(s))+\\ \left.\frac{1}{2}a_\varepsilon(\xi_\varepsilon(s),\eta_\varepsilon(s))\ddot{\gamma}_\varepsilon(\xi_\varepsilon(s))\right\}ds+\varepsilon^\delta\int_r^t g_\varepsilon(\xi_\varepsilon(s),\eta_\varepsilon(s))\dot{\gamma}_\varepsilon(\xi_\varepsilon(s))ds+\\ \varepsilon^\delta\int_r^t\sigma_\varepsilon(\xi_\varepsilon(s),\eta_\varepsilon(s))\dot{\gamma}_\varepsilon(\xi_\varepsilon(s))dw(s).\end{gathered} \tag{2.21}$$

By virtue the Lemma 3.7 for $h(x,y)=b(x,y)-\overline{b}(x)$ and $H(x)=\dot{\gamma}(x)$ we obtain

$$\int_r^t b_\varepsilon(\xi_\varepsilon(s),\eta_\varepsilon(s))\dot{\gamma}_\varepsilon(\xi_\varepsilon(s))ds=\int_r^t\overline{b}_\varepsilon(\xi_\varepsilon(s))\dot{\gamma}_\varepsilon(\xi_\varepsilon(s))ds+Q(b_\varepsilon-\overline{b}_\varepsilon,r,t). \tag{2.22}$$

By similar way for $h(x,y)=a(x,y)-\overline{a}(x)$ and $H(x)=\ddot{\gamma}(x)$ we arrive at

$$\int_r^t a_\varepsilon(\xi_\varepsilon(s),\eta_\varepsilon(s))\ddot{\gamma}_\varepsilon(\xi_\varepsilon(s))ds=\int_r^t\overline{a}_\varepsilon(\xi_\varepsilon(s))\ddot{\gamma}_\varepsilon(\xi_\varepsilon(s))ds+Q(a_\varepsilon-\overline{a}_\varepsilon,r,t). \tag{2.23}$$

Rewriting (2.21) and taking into account the equality $\overline{L}_x\gamma(x) = \frac{1}{\alpha}F(x)\overline{g}(x) - \beta_0$ and the relations (2.22), (2.23), we have

$$\begin{gathered}\int_r^t \left\{\frac{1}{\alpha}F_\varepsilon(\xi_\varepsilon(s))\overline{g}_\varepsilon(\xi_\varepsilon(s)) - \beta_0\right\} = \varepsilon^{2\delta}\left(\gamma_\varepsilon(\xi_\varepsilon(t)) - \gamma_\varepsilon(\xi_\varepsilon(r))\right) - \\ \varepsilon^\delta \int_r^t g_\varepsilon(\xi_\varepsilon(s), \eta_\varepsilon(s))\dot{\gamma}_\varepsilon(\xi_\varepsilon(s))ds - \varepsilon^\delta \int_r^t \sigma_\varepsilon(\xi_\varepsilon(s), \eta_\varepsilon(s))\dot{\gamma}_\varepsilon(\xi_\varepsilon(s))dw(s) - \\ Q(b_\varepsilon - \overline{b}_\varepsilon, r, t) - \frac{1}{2}Q(a_\varepsilon - \overline{a}_\varepsilon, r, t)\end{gathered} \tag{2.24}$$

We would like to estimate the right hand side of (2.24). Using Lemma 3.6 and Lemma 3.8 (see (3.23)), we obtain (2.19) and, consequently, (2.18).

By similar way we can prove

$$\lim_{\varepsilon\to 0} I_\varepsilon^4 = 0. \tag{2.25}$$

This result can be obtained by an application Ito formula to the function $\varepsilon^\delta\gamma^1(\xi_\varepsilon(t))$, where

$$\gamma^1(x) = 2\int_0^x F(z)\int_0^z \frac{\alpha^{-2}F^2(y)\overline{a}(y) - \sigma_0^2}{F(y)\overline{a}(y)}dydz. \tag{2.26}$$

After that we use the statements of Lemma 3.7 and Lemma 3.8.

Using (2.16),(2.17),(2.18), and (2.25), for the right hand side of (2.15) we arrive at

$$\lim_{\varepsilon\to 0}\{I_\varepsilon^0 + I_\varepsilon^1 + I_\varepsilon^2 + I_\varepsilon^3 + I_\varepsilon^4\} = 0. \tag{2.27}$$

Let μ_δ denotes some limit point of the family $\{\mu_\delta^\varepsilon, 0 < \varepsilon < \varepsilon_0\}$ and $\mathbf{E}^{\mu_\delta}$ be an expectation on this measure. Let come to the limit in (2.15) by the subsequence $\{\varepsilon_k\}$ such that $\mu_\delta^{\varepsilon_k} \Rightarrow \mu_\delta$ as $\varepsilon_k \to 0$. Taking into account (2.27), we get

$$\mathbf{E}^{\mu_\delta}\Phi_r(\xi)\left[\phi(\xi(t)) - \phi(\xi(r)) - \int_r^t \{\dot{\phi}(\xi(s))\beta_0 + \frac{1}{2}\ddot{\phi}(\xi(s))\sigma_0^2\}ds\right] = 0.$$

The coefficients does not depend on δ. That means $\mu_\delta = \mu$. Consequently, $\mu_\delta^\varepsilon \Rightarrow \mu$ as $\varepsilon \to 0$ and limit measure coinsides with the measure corresponding to the process (1.7). □

3.Preliminary results

In this section we prove the results used for the proof of the Theorem above.

The assertions of the next lemma follow from our assumptions.

Lemma 3.1. *Let the conditions (B) are satisfied. Then there exists the positive constant C such that*

a) $\exp\{-2\lambda_2\} \le |F(x)| \le \exp\{2\lambda_2\}$; $|\dot{F}(x)| \le C$; $|\ddot{F}(x)| \le C$; $|\dddot{F}(x)| \le C$;

b) $|h(x)| \le \exp\{2\lambda_2\}|x|$; $|\dot{h}(x)| \le C$;

c) $|f(x)| \le C(1 + |x|)$; $|\dot{f}(x)| \le C$.

Lemma 3.2. *Let the conditions (A) and (B) are satisfied. For every integer positive* m *and* $\delta \in]0, \frac{1}{3}[$ *and* $\varepsilon_0 > 0$ *there exist the constants* $C_m(\varepsilon_0)$*, such that for every* $\varepsilon < \varepsilon_0$

$$\mathbf{E} \sup_{t\in[0,T]} |\xi_\varepsilon(t)|^m \le C_m(\varepsilon_0)(1 + |\xi_0|)^m.$$

Proof. We note that

$$\varepsilon^\delta \tilde{L}_x^\varepsilon h_\varepsilon(x) = \alpha\varepsilon^\delta \tilde{L}_x^\varepsilon \left(f_\varepsilon(x) + \frac{x}{\varepsilon^\delta} \right) =$$
$$\alpha\varepsilon^{-\delta} \left[L_x^\varepsilon f_\varepsilon(x) + b_\varepsilon(x,y)\right] = \alpha\varepsilon^{-\delta} \left[b_\varepsilon^0(x,y) + b_\varepsilon(x,y)\right] = \alpha\varepsilon^{-\delta} B_\varepsilon(x,y),$$

the operator L_x^ε was defined by (1.6). We fix arbitrary $\varepsilon_0 > 0$ and for $\varepsilon < \varepsilon_0$ apply Ito formula to the function $\varepsilon^\delta h_\varepsilon(\xi_\varepsilon(t))$. We get

$$\begin{gathered}\varepsilon^\delta[h_\varepsilon(\xi_\varepsilon(t)) - h_\varepsilon(\xi_0)] = \alpha\varepsilon^{-\delta} \int_0^t B_\varepsilon(\xi_\varepsilon(s), \eta_\varepsilon(s))ds + \\ \int_0^t \dot{h}_\varepsilon(\xi_\varepsilon(s)) g_\varepsilon(\xi_\varepsilon(s), \eta_\varepsilon(s))ds + \int_0^t \dot{h}_\varepsilon(\xi_\varepsilon(s))\sigma_\varepsilon(\xi_\varepsilon(s), \eta_\varepsilon(s))dw(s).\end{gathered} \tag{3.1}$$

Taking into account (2.3), from (2.4) we get

$$\begin{gathered}\varepsilon^{-\delta} \int_0^t B_\varepsilon(\xi_\varepsilon(s), \eta_\varepsilon(s))ds = \varepsilon^{1-\delta}[m_\varepsilon(\xi_\varepsilon(t), \eta_\varepsilon(t)) - m_\varepsilon(\xi_0, \eta_0)] - \\ \int_0^t A_0(\varepsilon, s)ds - \int_0^t A_1(\varepsilon, s)ds - \int_0^t A_2(\varepsilon, s)dw(s) - \int_0^t A_3(\varepsilon, s)dw_1(s).\end{gathered} \tag{3.2}$$

Under our condition (B) and Lemma 3.1 from (3.1) and (3.2), taking into account (2.5) and (2.6), the process

$$\varepsilon^\delta h_\varepsilon(\xi_\varepsilon(t)) - \alpha\varepsilon^{1-\delta} m_\varepsilon(\xi_\varepsilon(t), \eta_\varepsilon(t))$$

has uniformly bounded coefficients. By the standard estimates as [4, Ch.II,sec.5], taking into account that $m(x,y)$ is uniformly bounded function, we obtain

$$\mathbf{E} \sup_{t\in[0,T]} |\varepsilon^\delta h_\varepsilon(\xi_\varepsilon(t))|^m \le C_m(1 + C_\varepsilon)(1 + |\xi_0|^m) \le C_m(\varepsilon_0)(1 + |\xi_0|^m),$$

here $\lim_{\varepsilon\to 0} C_\varepsilon = 0$. Using the properties of the function $\varepsilon^\delta h_\varepsilon(x)$, as in [6, Lemma 2.5, p.218], we obtain the statements of the lemma. ☐

Lemma 3.3. *Let the functions* $H(x) \in C_{x,b}^2(E_1)$ *and the function* $n(x,y) \in C_{x,y,b}^{2,2}(E_1, E_n)$ *is satisfied the condition* $\bar{n}(x) = 0$*. The processes* $\xi_\varepsilon(t), \eta_\varepsilon(t)$ *are the solutions of (1.1), (1.2) respectively,* $\zeta_\varepsilon(t)$ *is defined by (2.7). Then for a function* $\psi(x) \in C_0^\infty$ *and a continuous bounded* $\mathcal{C}_s$ *— measurable functional* $\Phi_s(x)$

$$\lim_{\varepsilon\to 0} \mathbf{E}\Phi_r(x) \left[\int_r^t \psi(\zeta_\varepsilon(s)) H_\varepsilon(\xi_\varepsilon(s)) n_\varepsilon(\xi_\varepsilon(s), \eta_\varepsilon(s))ds \right] = 0.$$

Proof. Let the function $l(x,y)$ be the unique solution of the problem

$$Ll(x,y) = n(x,y), \qquad \int_Y l(x,y)dy = 0$$

for any $x \in E_1$ (x play a role of parameter), because $\bar{n}(x) = 0$. Then $l(x,y) \in C^{2,2}_{x,y,b}(E_1, E_n)$. Let $l_\varepsilon(s) = l\Big(\frac{\xi_\varepsilon(s)}{\varepsilon^\delta}, \eta_\varepsilon(s)\Big)$ and the same sense have the denotations $g_\varepsilon(s), \sigma_\varepsilon(s), a_\varepsilon(s)$. Applying Ito formula to the function

$$\varepsilon\psi(\zeta_\varepsilon(s))H_\varepsilon(\xi_\varepsilon(s))l\bigg(\frac{\xi_\varepsilon(s)}{\varepsilon^\delta}, \eta_\varepsilon(s)\bigg),$$

we get

$$\begin{gathered}
\varepsilon\left[\psi(\zeta_\varepsilon(t))H_\varepsilon(\xi_\varepsilon(t))l_\varepsilon(t) - \psi(\zeta_\varepsilon(r))H_\varepsilon(\xi_\varepsilon(r))l_\varepsilon(r)\right] = \\
\varepsilon\int_r^t \dot{\psi}(\zeta_\varepsilon(s))H_\varepsilon(\xi_\varepsilon(s))\left[l_\varepsilon(s)\left(\alpha^{-1}F_\varepsilon(\xi_\varepsilon(s))g_\varepsilon(s) + A_0(\varepsilon,s) + A_1(\varepsilon,s)\right) + \right. \\
\left.\frac{1}{\sqrt{\varepsilon}}A_3(\varepsilon,s)\nabla_y l_\varepsilon(s)\sigma_1\left(\eta_\varepsilon(s)\right)\right] ds + \frac{\varepsilon}{2}\int_r^t \ddot{\psi}(\zeta_\varepsilon(s))H_\varepsilon(\xi_\varepsilon(s))l_\varepsilon(s)\times \\
\left[\{\alpha^{-1}F_\varepsilon(\xi_\varepsilon(s))\sigma_\varepsilon(s) + A_2(\varepsilon,s)\}^2 + A_5^2(1,\varepsilon,\xi_\varepsilon(s))\right] ds + \\
\varepsilon\int_r^t \dot{\psi}(\zeta_\varepsilon(s))H_\varepsilon(\xi_\varepsilon(s))l_\varepsilon(s)\left(\alpha^{-1}F_\varepsilon(\xi_\varepsilon(s))\sigma_\varepsilon(s) + A_2(\varepsilon,s)\right) dw(s) + \\
\varepsilon\int_r^t \dot{\psi}(\zeta_\varepsilon(s))H_\varepsilon(\xi_\varepsilon(s))l_\varepsilon(s)A_3(\varepsilon,s)dw_1(s) + \\
\varepsilon\int_r^t \psi(\zeta_\varepsilon(s))\left(\dot{H}_e(\xi_\varepsilon(s))l_\varepsilon(s) + \frac{1}{\varepsilon^\delta}H_\varepsilon(\xi_\varepsilon(s))\frac{\partial l_\varepsilon(s)}{\partial x}\right)\left(\frac{1}{\varepsilon^\delta}b(\xi_\varepsilon) + g_\varepsilon(s)\right) ds + \\
\frac{\varepsilon}{2}\int_r^t \psi(\zeta_\varepsilon(s))\left[\ddot{H}_\varepsilon(\xi_\varepsilon(s))l_\varepsilon(s) + \frac{2}{\varepsilon^\delta}\dot{H}_\varepsilon(\xi_\varepsilon(s))\frac{\partial l_\varepsilon(s)}{\partial x} + \right. \\
\left.\frac{1}{\varepsilon^{2\delta}}H_\varepsilon(\xi_\varepsilon(s))\frac{\partial^2 l_\varepsilon(s)}{\partial x^2}\right] a_\varepsilon(s)ds + \varepsilon\int_r^t \psi(\zeta_\varepsilon(s))\left(\dot{H}_\varepsilon(\xi_\varepsilon(s)))l_\varepsilon(s) + \right. \\
\left.\frac{1}{\varepsilon^\delta}H_\varepsilon(\xi_\varepsilon(s))\frac{\partial l_\varepsilon(s)}{\partial x}\right)\sigma_\varepsilon(s)dw(s) + \varepsilon\int_r^t \dot{\psi}(\zeta_\varepsilon(s))\left(\dot{H}_\varepsilon(\xi_\varepsilon(s)))l_\varepsilon(s) + \right. \\
\left.\frac{1}{\varepsilon^\delta}H_\varepsilon(\xi_\varepsilon(s))\frac{\partial l_\varepsilon(s)}{\partial x}\right)\left(\alpha^{-1}F_\varepsilon(\xi_\varepsilon(s))a_\varepsilon(s) + A_2(\varepsilon,s)\sigma_\varepsilon(s)\right) ds + \\
\int_r^t \psi(\zeta_\varepsilon(s))H_\varepsilon(\xi_\varepsilon(s))Ll_\varepsilon(s)ds + \\
\sqrt{\varepsilon}\int_r^t \psi(\zeta_\varepsilon(s))H_\varepsilon(\xi_\varepsilon(s))\nabla_y l_\varepsilon(s)\sigma_1\left(\frac{s}{\varepsilon}, \eta_\varepsilon(s)\right) dw_1(s).
\end{gathered}$$

From this we arrive at

$$\int_r^t \psi(\zeta_\varepsilon(s))H_\varepsilon(\xi_\varepsilon(s))n_\varepsilon(\xi_\varepsilon(s), \eta_\varepsilon(s))ds = G_\varepsilon(n,r,t), \tag{3.3}$$

where

$$\begin{gathered}
G_\varepsilon(n,r,t) = B_1(\varepsilon,t) - B_1(\varepsilon,r) \\
\int_r^t B_2(\varepsilon,s)ds + \int_r^t B_3(\varepsilon,s)dw(s) + \int_r^t B_4(\varepsilon,s)dw_1(s),
\end{gathered}$$

and here

$$
\begin{gathered}
B_1(\varepsilon,t)=\varepsilon\psi(\zeta_\varepsilon(t))H_\varepsilon(\xi_\varepsilon(t))l_\varepsilon(t);\\
B_2(\varepsilon,s)=-\Big[\varepsilon\dot{\psi}(\zeta_\varepsilon(s))H_\varepsilon(\xi_\varepsilon(s))\left[l_\varepsilon(s)\times\right.\\
\left.\left[\alpha^{-1}F_\varepsilon(\xi_\varepsilon(s))g_\varepsilon(s)+A_0(\varepsilon,s)+A_1(\varepsilon,s)\right]+\frac{1}{\sqrt{\varepsilon}}A_3(\varepsilon,s)\nabla_y l_\varepsilon(s)\sigma_1\left(\eta_\varepsilon(s)\right)\right]+\\
\frac{\varepsilon}{2}\ddot{\psi}(\zeta_\varepsilon(s))H_\varepsilon(\xi_\varepsilon(s))l_\varepsilon(s)\left[\{\alpha^{-1}F_\varepsilon(\xi_\varepsilon(s))\sigma_\varepsilon(s)+A_2(\varepsilon,s)\}^2+A_3^2(\varepsilon,s)\right]+\\
\varepsilon\psi(\zeta_\varepsilon(s))\left(\dot{H}_e(\xi_\varepsilon(s))l_\varepsilon(s)+\frac{1}{\varepsilon^\delta}H_\varepsilon(\xi_\varepsilon(s))\frac{\partial l_\varepsilon(s)}{\partial x}\right)\left(\frac{1}{\varepsilon^\delta}b(\xi_\varepsilon)+g_\varepsilon(s)\right)+\\
\frac{\varepsilon}{2}\psi(\zeta_\varepsilon(s))\left[\ddot{H}_\varepsilon(\xi_\varepsilon(s))l_\varepsilon(s)+\frac{2}{\varepsilon^\delta}\dot{H}_\varepsilon(\xi_\varepsilon(s))\frac{\partial l_\varepsilon(s)}{\partial x}+\frac{1}{\varepsilon^{2\delta}}H_\varepsilon(\xi_\varepsilon(s))\frac{\partial l_\varepsilon(s)}{\partial x}\right]a_\varepsilon(s)+\\
\varepsilon\dot{\psi}(\zeta_\varepsilon(s))\left(\dot{H}_\varepsilon(\xi_\varepsilon(s)))l_\varepsilon(s)+\frac{1}{\varepsilon^\delta}H_\varepsilon(\xi_\varepsilon(s))\frac{\partial l_\varepsilon(s)}{\partial x}\right)\times\\
\left(\alpha^{-1}F_\varepsilon(\xi_\varepsilon(s))a_\varepsilon(s)+A_2(\varepsilon,s)\sigma_\varepsilon(s)\right)\Big];\\
B_3(\varepsilon,s)=-\Big[\varepsilon\dot{\psi}(\zeta_\varepsilon(s))H_\varepsilon(\xi_\varepsilon(s))l_\varepsilon(s)\left(\alpha^{-1}F_\varepsilon(\xi_\varepsilon(s))\sigma_\varepsilon(s)+A_2(\varepsilon,s)\right)+\\
\varepsilon\psi(\zeta_\varepsilon(s))\left(\dot{H}_\varepsilon(\xi_\varepsilon(s)))l_\varepsilon(s)+\frac{1}{\varepsilon^\delta}H_\varepsilon(\xi_\varepsilon(s))\frac{\partial l_\varepsilon(s)}{\partial x}\right)\sigma_\varepsilon(s)\Big];\\
B_4(\varepsilon,s)=-\Big[\varepsilon\dot{\psi}(\zeta_\varepsilon(s))H_\varepsilon(\xi_\varepsilon(s))l_\varepsilon(s)A_3(\varepsilon,s)+\\
\sqrt{\varepsilon}\psi(\zeta_\varepsilon(s))H_\varepsilon(\xi_\varepsilon(s))\nabla_y l_\varepsilon(s)\sigma_1\left(\frac{s}{\varepsilon},\eta_\varepsilon(s)\right)\Big].
\end{gathered}
$$

Under our conditions we can obtain the estimates

$$
\begin{gathered}
|B_1(\varepsilon,s)|\le\varepsilon C(1+|\xi_\varepsilon|+|\xi_\varepsilon|^2);\quad |B_2(\varepsilon,s)|\le\varepsilon^{1-2\delta}C(1+\sum_{k=1}^{3}|\xi_\varepsilon|^k);\\
|B_3(\varepsilon,s)|\le\varepsilon^{1-\delta}C(1+\sum_{k=1}^{3}|\xi_\varepsilon|^k);\quad |B_4(\varepsilon,s)|\le\sqrt{\varepsilon}C(1+\sum_{k=1}^{3}|\xi_\varepsilon|^k).
\end{gathered}
$$

Hence, using the estimates of Lemma 3.2, from above by the standard arguments we arrive at

$$\lim_{\varepsilon\to0}\mathbf{E}\Phi_r(\xi_\varepsilon)G_\varepsilon(n,r,t)=0.$$

Letting $\varepsilon\to0$ in (3.3), we obtain the statement of lemma. □

Lemma 3.4. *Let the conditions (A), (B), and (C0) are hold. Then for every* $\delta\in]0;\frac{1}{3}[$

$$\lim_{\varepsilon\to0}\mathbf{E}\sup_{t\in[0,T]}|\varepsilon^\delta f_\varepsilon(\xi_\varepsilon(t))-\varepsilon^{1-\delta}m_\varepsilon(\xi_\varepsilon(t),\eta_\varepsilon(t))|^2=0.$$

Proof. We have

$$\mathbf{E}\sup_{t\in[0,T]}|\varepsilon^\delta f_\varepsilon(\xi_\varepsilon(t))-\varepsilon^{1-\delta}m_\varepsilon(\xi_\varepsilon(t),\eta_\varepsilon(t))|^2\le2(D_\varepsilon^1+D_\varepsilon^2),\tag{3.4}$$

where

$$D^1_\varepsilon = \mathbf{E}\sup_{t\in[0,T]} |\varepsilon^\delta f_\varepsilon(\xi_\varepsilon(t))|^2, \qquad D^2_\varepsilon = \mathbf{E}\sup_{t\in[0,T]} |\varepsilon^{1-\delta} m_\varepsilon(\xi_\varepsilon(t),\eta_\varepsilon(t))|^2.$$

For every $\varepsilon > 0$ and $\varepsilon^{\frac{\delta}{2}} < N < \infty$

$$\begin{aligned} D^1_\varepsilon &= \mathbf{E}\sup_{t\in[0,T]} |\varepsilon^\delta f_\varepsilon(\xi_\varepsilon(t))|^2 \chi\{|\xi_\varepsilon(t)| < \varepsilon^{\frac{\delta}{2}}\} + \\ &\mathbf{E}\sup_{t\in[0,T]} |\varepsilon^\delta f_\varepsilon(\xi_\varepsilon(t))|^2 \chi\{\varepsilon^{\frac{\delta}{2}} \le |\xi_\varepsilon(t)| \le N\} + \\ &\mathbf{E}\sup_{t\in[0,T]} |\varepsilon^\delta f_\varepsilon(\xi_\varepsilon(t))|^2 \chi\{|\xi_\varepsilon(t)| > N\} = D^{11}_\varepsilon + D^{12}_\varepsilon + D^{13}_\varepsilon \end{aligned} \tag{3.5}$$

respectively. By property c) of Lemma 3.1 we obtain

$$\lim_{\varepsilon\to 0} D^{11}_\varepsilon = 0. \tag{3.6}$$

Using the definition of the function $f(x)$ and the condition (C0), we have

$$\lim_{\varepsilon\to 0} D^{12}_\varepsilon = \lim_{\varepsilon\to 0}\mathbf{E}\sup_{t\in[0,T]} \left| \frac{\xi_\varepsilon(t)}{\alpha}\frac{\varepsilon^\delta}{\xi_\varepsilon(t)} \int_0^{\frac{\xi_\varepsilon(t)}{\varepsilon^\delta}} F(z)dz - \xi_\varepsilon(t)\right|^2 \chi\{\varepsilon^{\frac{\delta}{2}} \le |\xi_\varepsilon(t)| \le N\} = 0. \tag{3.7}$$

Now

$$D^{13}_\varepsilon \le \varepsilon^{2\delta} C \left(1 + \frac{\mathbf{E}\sup_{t\in[0,T]} |\xi_\varepsilon(t)|^4}{\varepsilon^{4\delta}}\right)^{\frac{1}{2}} \frac{\mathbf{E}\sup_{t\in[0,T]} |\xi_\varepsilon(t)|^2}{N^2}.$$

At first approaching the limit as $\varepsilon \to 0$ and then as $N \to \infty$ and using the estimation of Lemma 3.2, we obtain

$$\lim_{\varepsilon\to 0} D^{13}_\varepsilon = 0. \tag{3.8}$$

From (3.5)–(3.8) we conclude that

$$\lim_{\varepsilon\to 0} D^{1}_\varepsilon = 0. \tag{3.9}$$

From the definition of $m(\xi_\varepsilon(t), \eta_\varepsilon(t))$ follows the existence of the positive constant C such that

$$\lim_{\varepsilon\to 0} D^{2}_\varepsilon \le \lim_{\varepsilon\to 0} \varepsilon^{2(1-\delta)} C = 0. \tag{3.10}$$

Now, from (3.4), (3.9), and (3.10) follows the statement of lemma. □

Lemma 3.5. *Let the conditions (A) and (B) are hold, and let $\psi_\varepsilon(x)$ be such a function that*

$$\mathbf{E}\sup_{t\in[0,T]} |\psi_\varepsilon(\xi_\varepsilon(t))|^2 \le K \tag{3.11}$$

and for every $r, t : 0 \le r \le t \le T$

$$\lim_{\varepsilon \to 0} \mathbf{E} \left| \int_r^t \psi_\varepsilon(\xi_\varepsilon(s)) ds \right| = 0. \tag{3.12}$$

Then for every $\phi(x) \in C_0^\infty$ *and* $0 \le r \le t \le T$

$$\lim_{\varepsilon \to 0} \mathbf{E} \left| \int_r^t \phi(\xi_\varepsilon(s)) \psi_\varepsilon(\xi_\varepsilon(s)) ds \right| = 0.$$

Proof. Let $\{t_i\}$ be some partition of interval $[r, t] : r \le t_1 \le t_2 \le ... \le t_n = t$ such that $|t_{i+1} - t_i| \le \eta_n$ and $\lim_{n \to \infty} \eta_n = 0$. Then

$$\begin{aligned} \mathbf{E} \left| \int_r^t \phi(\xi_\varepsilon(s)) \psi_\varepsilon(\xi_\varepsilon(s)) ds \right| \le \mathbf{E} \sum_{i=1}^{n-1} \left| \int_{t_i}^{t_{i+1}} \Big[\phi(\xi_\varepsilon(s)) - \phi(\xi_\varepsilon(t_i)) \Big] \psi_\varepsilon(\xi_\varepsilon(s)) ds \right| + \\ \sum_{i=1}^{n-1} \mathbf{E} \Big| \phi(\xi_\varepsilon(t_i)) \Big| \left| \int_{t_i}^{t_{i+1}} \psi_\varepsilon(\xi_\varepsilon(s)) ds \right|. \end{aligned} \tag{3.13}$$

We want to estimate the first term in the right hand side (3.13). Let us denote this term by $L(\varepsilon, n)$, then, by using the estimation (3.11), there exists the constant C_0, C_ε such that

$$L(\varepsilon, n) \le \sum_{i=1}^{n-1} \left(\int_{t_i}^{t_{i+1}} \mathbf{E} \Big| \phi(\xi_\varepsilon(s)) - \phi(\xi_\varepsilon(t_i)) \Big|^4 ds \right)^{\frac{1}{4}} \left(\int_{t_i}^{t_{i+1}} \mathbf{E} \Big| \psi_\varepsilon(\xi_\varepsilon(s)) \Big|^{\frac{4}{3}} ds \right)^{\frac{3}{4}} \le$$
$$C_0 \left\{ (1 + C_\varepsilon) \sum_{i=1}^{n-1} (t_{i+1} - t_i)^{\frac{3}{2}} + \varepsilon^{1-\delta} \sum_{i=1}^{n-1} (t_{i+1} - t_i) \right\} \le C_0 (t - r) \left\{ (1 + C_\varepsilon) \eta_n^{\frac{1}{2}} + \varepsilon^{1-\delta} \right\},$$

and $\lim_{\varepsilon \to 0} C_\varepsilon = 0$. Consequently, the first term in (3.13) can be made sufficiently small by making the partition of the interval $[r, t]$ fine enough. From (3.12) the second term in (3.13) tends to 0 as $\varepsilon \to 0$. The lemma is proved. $\square$

Lemma 3.6. *Let the conditions (A), (B), and (C1) be fulfilled. The function* $\gamma(x)$ *defined by (2.20),* $\xi_\varepsilon(t)$ *is the solution of (1.1). Then for every* $\delta \in]0; \frac{1}{3}[$

$$\lim_{\varepsilon \to 0} \mathbf{E} \sup_{t \in [0,T]} |\varepsilon^{2\delta} \gamma_\varepsilon(\xi_\varepsilon(t))| = \lim_{\varepsilon \to 0} \mathbf{E} \sup_{t \in [0,T]} |\varepsilon^{\delta} \dot{\gamma}_\varepsilon(\xi_\varepsilon(t))|^2 = 0.$$

Proof. For $\gamma(x)$, according to Lemma 3.1, under the condition (B) we have

$$|\gamma(x)| \le C(1 + |x|^2); \qquad \sum_{i=1}^{4} \Big| \frac{d^i}{dx^i} \gamma(x) \Big| \le C(1 + |x|). \tag{3.14}$$

Since for every $N < \infty$ under the condition (C)

$$\lim_{\varepsilon \to 0} \sup_{0 < |x| \le N} \varepsilon^\delta \int_0^{\frac{x}{\varepsilon^\delta}} \frac{\alpha^{-1} F(y) \overline{g}(y) - \beta_0}{F(y) \overline{a}(y)} dy =$$
$$\lim_{\varepsilon \to 0} \sup_{0 < |x| \le N} x \left(\frac{\varepsilon^\delta}{x} \int_0^{\frac{x}{\varepsilon^\delta}} \frac{\overline{g}(y)}{\alpha \overline{a}(y)} dy - \frac{\varepsilon^\delta}{x} \int_0^{\frac{x}{\varepsilon^\delta}} \frac{\beta_0}{F(y) \overline{a}(y)} dy \right) = 0,$$

then by property a) of Lemma 3.1 we obtain

$$\lim_{\varepsilon\to 0}\sup_{0<|x|\le N}|\varepsilon^{2\delta}\gamma_\varepsilon(x)| = \lim_{\varepsilon\to 0}\sup_{0<|x|\le N}|\varepsilon^{\delta}\dot\gamma_\varepsilon(x)|^2 = 0 \tag{3.15}$$

Now, for every $N : \varepsilon^{\frac{\delta}{2}} \le N < \infty$,

$$\begin{aligned}\mathbf{E}\sup_{t\in[0,T]}|\varepsilon^{2\delta}\gamma_\varepsilon(\xi_\varepsilon(t))| = \lim_{\varepsilon\to 0}\mathbf{E}\sup_{t\in[0,T]}|\varepsilon^{2\delta}\gamma_\varepsilon(\xi_\varepsilon(t))|\Big(\chi\{|\xi_\varepsilon(t)|<\varepsilon^{\frac{\delta}{2}}\}+ \\ \chi\{\varepsilon^{\frac{\delta}{2}}\le|\xi_\varepsilon(t)|\le N\}+\chi\{|\xi_\varepsilon(t)|>N\}\Big) = \gamma_\varepsilon^1+\gamma_\varepsilon^2+\gamma_\varepsilon^3\end{aligned} \tag{3.16}$$

respectively. By inequality (3.14) we have

$$\lim_{\varepsilon\to 0}\gamma_\varepsilon^1 \le \lim_{\varepsilon\to 0}\varepsilon^{2\delta}C(1+\varepsilon^{\frac{\delta^2}{4}-2\delta}) = 0. \tag{3.17}$$

Using (3.15), we obtain

$$\lim_{\varepsilon\to 0}\gamma_\varepsilon^2 = 0. \tag{3.18}$$

Taking into account (3.14) and the result of Lemma 3.2, we get

$$\gamma_\varepsilon^3 \le \varepsilon^{2\delta}C(1+\varepsilon^{-4\delta}\mathbf{E}\sup_{t\in[0,T]}|\xi_\varepsilon(t)|^4)^{\frac{1}{2}}\frac{\mathbf{E}\sup_{t\in[0,T]}|\xi_\varepsilon(t)|^2}{N^2} \le \frac{C}{N^2}.$$

Approaching the limit as $\varepsilon\to 0$ and then as $N\to\infty$, we obtain

$$\lim_{\varepsilon\to 0}\gamma_\varepsilon^3 = 0. \tag{3.19}$$

From (3.16)–(3.19) follows

$$\lim_{\varepsilon\to 0}\mathbf{E}\sup_{t\in[0,T]}|\varepsilon^{2\delta}\gamma_\varepsilon(\xi_\varepsilon(t))| = 0.$$

By similar way we can show that

$$\lim_{\varepsilon\to 0}\mathbf{E}\sup_{t\in[0,T]}|\varepsilon^{\delta}\dot\gamma_\varepsilon(\xi_\varepsilon(t))|^2 = 0.$$

□

By similar way we can prove the next statement.

Lemma 3.7. *Let the conditions (A), (B), and (C2) be fulfilled. The function $\gamma^1(x)$ is defined by (2.26), $\xi_\varepsilon(t)$ is the solution of (1.1). Then for every $\delta\in]0;\frac{1}{3}[$*

$$\lim_{\varepsilon\to 0}\mathbf{E}\sup_{t\in[0,T]}|\varepsilon^{2\delta}\gamma_\varepsilon^1(\xi_\varepsilon(t))| = \lim_{\varepsilon\to 0}\mathbf{E}\sup_{t\in[0,T]}|\varepsilon^{\delta}\dot\gamma_\varepsilon^1(\xi_\varepsilon(t))|^2 = 0.$$

Lemma 3.8. *Let the functions* $H(x) \in C^2_x(E_1)$ *such that* $|H(x)| + |\dot{H}(x)| + |\ddot{H}(x)| \leq C(1 + |x|)$ *and periodic of period 1 with respect to y function* $h(x,y) \in C^{2,2}_{x,y,b}\ (E_1, E_n)$ *such that* $\bar{h}(x) = 0$*. The processes* $\xi_\varepsilon(t), \eta_\varepsilon(t)$ *are the solutions of (1.1), (1.2) respectively. Then for every* $\delta \in]0, \frac{1}{3}[$ *and* $0 \leq r \leq t \leq T$

$$\lim_{\varepsilon \to 0} \mathbf{E} \Big| \int_r^t H_\varepsilon(\xi_\varepsilon(s)) h\bigg(\frac{\xi_\varepsilon(s)}{\varepsilon^\delta}, \eta_\varepsilon(s)\bigg) ds \Big| = 0.$$

Proof. Let the function $l(x,y)$ be the solution of the problem

$$Ll(x,y) \;=\; h(x,y), \qquad \int_Y l(x,y)dy \;=\; 0 \tag{3.20}$$

for every $x \in E_1$ (x play a role of parameter), because $\bar{h}(x) = 0$. Then $l(x,y) \in C^{2,2}_{x,y,b}(E_1, E_n)$. Let $l_\varepsilon(s) = l\Big(\frac{\xi_\varepsilon(s)}{\varepsilon^\delta}, \eta_\varepsilon(s)\Big)$ and the same sense have the denotations $g_\varepsilon(s), \sigma_\varepsilon(s), a_\varepsilon(s)$. Applying Ito formula to the function

$$\varepsilon H_\varepsilon(\xi_\varepsilon(t)) l\bigg(\frac{\xi_\varepsilon(t)}{\varepsilon^\delta}, \eta_\varepsilon(t)\bigg)$$

and using (3.20), after the rearrangement of the terms we obtain

$$\int_r^t H_\varepsilon(\xi_\varepsilon(s)) h_\varepsilon(\xi_\varepsilon(s), \eta_\varepsilon(s)) ds = Q(h_\varepsilon, r, t),$$

where

$$\begin{gathered}
Q(h_\varepsilon, r, t) = \varepsilon\{H_\varepsilon(\xi_\varepsilon(t)) l_\varepsilon(t) - H_\varepsilon(\xi_\varepsilon(r)) l_\varepsilon(r)\} - \\
\varepsilon^{1-2\delta} \int_r^t \{L^\varepsilon_{\xi_\varepsilon(s)}(H_\varepsilon(\xi_\varepsilon(s)) l_\varepsilon(s)) + \dot{H}_\varepsilon(\xi_\varepsilon(s)) \frac{\partial l_\varepsilon(s)}{\partial x}\} ds - \\
\varepsilon^{1-\delta} \int_r^t g_\varepsilon(s) \frac{\partial}{\partial x}(H_\varepsilon(\xi_\varepsilon(s)) l_\varepsilon(s)) ds - \varepsilon^{1-\delta} \int_r^t \sigma_\varepsilon(s) \frac{\partial}{\partial x}(H_\varepsilon(\xi_\varepsilon(s)) l_\varepsilon(s)) dw(s) - \\
\varepsilon^{\frac{1}{2}} \int_r^t \sigma_1(\frac{s}{\varepsilon^\delta}, \eta_\varepsilon(s)) H_\varepsilon(\xi_\varepsilon(s)) \nabla_y l_\varepsilon(s) dw_1(s).
\end{gathered} \tag{3.21}$$

Under our conditions and by estimation of Lemma 3.2 all integrands in (3.21) can be estimated by the constant multiplied to ε in positive powers. For example,

$$\varepsilon^{1-2\delta} \mathbf{E} \sup_{t \in [0,T]} |L^\varepsilon_{\xi_\varepsilon(s)}(H_\varepsilon(\xi_\varepsilon(s)) l_\varepsilon(s))| \leq C\varepsilon^{1-3\delta},$$

and

$$\mathbf{E} \sup_{t \in [0,T]} \big| \varepsilon^{1-\delta} \sigma_\varepsilon(s) \frac{\partial}{\partial x}(H_\varepsilon(\xi_\varepsilon(s)) l_\varepsilon(s)) \big|^2 \leq C\varepsilon^{2(1-\delta)},$$

and so on. Hence, by standard way from (3.24) we obtain

$$\lim_{\varepsilon \to 0} \mathbf{E}|Q(h_\varepsilon, r, t)| = 0.$$

By the above relation (3.23) the proof is complete. □

4.Example

In this section the result of theorem is applied to one class of the stochastic processes.

Let $\eta_\varepsilon(t)$ is one-dimensional process (the solution of (1.2), when $n = 1$) and the condition (A) is satisfied. Then

$$L = \frac{1}{2}a^1(y)\frac{d^2}{dy^2} + g^1(y)\frac{d}{dy}$$

and the problem

$$L^*p(y) = 0, \qquad \int_0^1 p(y)dy = 1$$

has the solution

$$p(y) = \frac{V(y)}{C_0 a^1(y)},$$

where

$$V(y) = exp\left(\int_0^y \frac{2g^1(z)}{a_1(z)}dz\right), \text{ and } \qquad C_0 = \int_0^1 \frac{V(y)}{a^1(y)}dy.$$

We have

$$\overline{a}(x) = \int_0^1 a(x,y)p(y)dy, \quad \overline{b}(x) = \int_0^1 b(x,y)p(y)dy, \quad \overline{g}(x) = \int_0^1 g(x,y)p(y)dy.$$

If the conditions (B) and (C) are satisfied, then the limit process is

$$\xi(t) = \xi_0 + \frac{\alpha_2}{\alpha\alpha_1}t + \frac{1}{\alpha\alpha_1}w(t).$$

In this case all auxiliary functions have the explicit form.

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