Graph Theory and Combinatorics MATH-42021/52021.
Home Work 3, due on Saturday, June 26
Instructor: Prof. Artem Zvavitch
13 points (yes, 3 extra points!)

Problem 1. *Build* 6*-vertex graphs with the following degrees of vertices, if possible. If not possible, explain why not:*

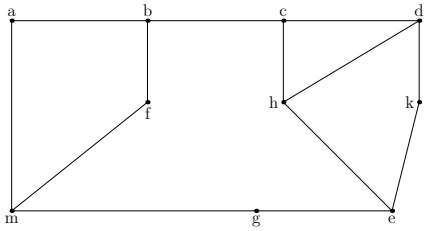
- $3, 3, 3, 1, 1, 1$.
- $1, 2, 2, 3, 4, 5$.
- $2, 2, 4, 4, 4, 4$.

Can you make your examples connected? Planar?

Problem 2. *Prove that every connected planar graph with less than* 12 *vertices has a vertex of degree at most* 4*.*

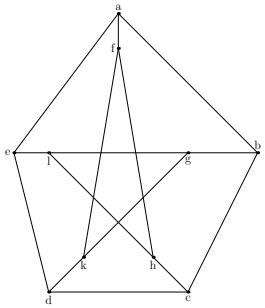
Problem 3. *Prove that if a connected graph has* $2k$ *vertices of odd degree, then there are* k *disjoint trails that contain all edges.*

Problem 4. *Consider the graph* G



- *Does this graph contain Euler cycle?*
- *Does this graph contain Euler trail?*

Problem 5. *Please, TRY to remove some edges from the graph below, in a such a way that a new graph would contain and Euler cycle (note that the graph must stay connected). Explain, if this is impossible* **(this part is Tricky!)**.



Problem 6. *In chess a "knight move" consists of two squares either vertically or horizontally and then one square in a perpendicular direction. Depending on where the knight is situated, he has a minimum mobility of two moves - when in a corner- and a maximum mobility of eight moves when near the center. Let* C *be a graph with* 64 *vertices corresponding to the squares of a chessboard. Let two vertices of* C *be joint by an edge whenever a knight can go from one of the correspond- ing squares to the other in the move. Does* C *have an Euler trail? (You don't have to draw* C *to answer!!!!)*