

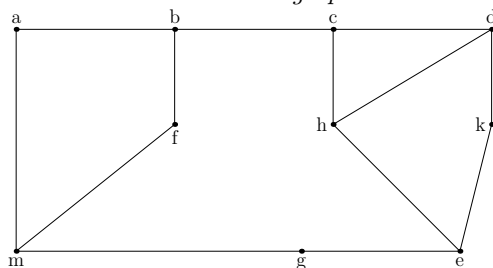
Graph Theory and Combinatorics MATH-42021/52021.

Home Work 4, due on Tuesday, June 28

Instructor: Prof. Artem Zvavitch

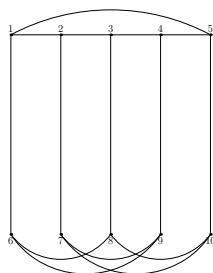
10 points (+ an extra problem for 5 points)

**Problem 1.** Consider the graph  $G$



- Does this graph contains Hamilton circuit?
- Does this graph contains Hamilton path?

**Problem 2.** Find a Hamiltonian path and prove that no Hamiltonian circuit exists for the following graph:



**Problem 3. (+ 5 points)** In chess a "knight move" consists of two squares either vertically or horizontally and then one square in a perpendicular direction. Depending on where the knight is situated, he has a minimum mobility of two moves - when in a corner- and a maximum mobility of eight moves when near the center. Let  $C$  be a graph with 64 vertices corresponding to the squares of a chessboard. Let two vertices of  $C$  be joint by an edge whenever a knight can go from one of the corresponding squares to the other in the move. Prove that the graph  $C$  in the above problem has Hamilton path. Such a path is called "knight's tour" by puzzle enthusiasts.

**Problem 4.** Please, answer Yes or No to the following questions (do not forget to provide explanation)

- Is it true that if  $G$  is isomorphic to  $G'$  then  $\chi(G) = \chi(G')$ ?
- Is it true that if  $\chi(G) = \chi(G')$ , then  $G$  is isomorphic to  $G'$ ?

**NEXT PAGE, PLEASE**

**Problem 5.** Find the chromatic number of each of the graphs

