

Graph Theory and Combinatorics MATH-42021/52021.

Home Work 9, due on Saturday, July 16

Instructor: Prof. Artem Zvavitch

6 problems, 2pts each, YES 2 points extra!

**Problem 1.** Prove that

$$\binom{2n}{2} = 2\binom{n}{2} + n^2.$$

**Problem 2.** Prove that

$$\binom{n}{1} + 6\binom{n}{2} + 6\binom{n}{3} = n^3.$$

**Problem 3.** Evaluate

$$1^3 + 2^3 + 3^3 + \dots + n^3.$$

**Problem 4.** Evaluate

$$\sum_{k=1}^n k3^k \binom{n}{k}.$$

**Problem 5.** Evaluate  $\sum_{k=0}^n k(n-k)$ .

**Problem 6.** Show that

$$\sum_{k=0}^n \frac{(2n)!}{(k!)^2((n-k)!)^2} = \binom{2n}{n}^2.$$