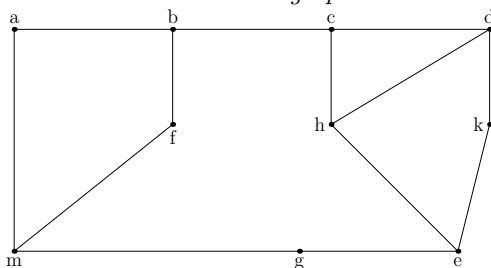**Graph Theory and Combinatorics MATH-42021/52021.**
Home Work 4, due on Tuesday, July 31
Instructor: Prof. Artem Zvavitch
10 points (+ an extra problem for 5 points)

Problem 1. *Consider the graph G*

- *Does this graph contains Hamilton circuit?*
- *Does this graph contains Hamilton path?*

Problem 2. *Find a Hamiltonian path and prove that no Hamiltonian circuit exists for the following graph:*
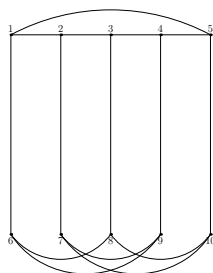
Problem 3. (+ 5 points) *In chess a "knight move" consists of two squares either vertically or horizontally and then one square in a perpendicular direction. Depending on where the knight is situated, he has a minimum mobility of two moves - when in a corner- and a maximum mobility of eight moves when near the center. Let C be a graph with 64 vertices corresponding to the squares of a chessboard. Let two vertices of C be joint by an edge whenever a knight can go from one of the corresponding squares to the other in the move. Prove that the graph C in the above problem has Hamilton path. Such a path is called "knight's tour" by puzzle enthusiasts.*

Problem 4. *Please, answer Yes or No to the following questions (do not forget to provide explanation)*

- *Is it true that if G is isomorphic to G' then $\chi(G) = \chi(G')$?*
- *Is it true that if $\chi(G) = \chi(G')$, then G is isomorphic to G'?*

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Problem 5. *Find the chromatic number of each of the graphs*