Graph Theory and Combinatorics MATH-42021/52021. Home Work 7, due on Saturday, August 11 Instructor: Prof. Artem Zvavitch 8 problems, 2pts each, YES 6 points extra!

Problem 1. How many different sequences of heads and tails are possible if a coin is fllipped n times?

Problem 2. How many ways there to pick a man and a woman who are not husband and wife from a group of n married couples?

Problem 3. How many different six digit numbers can be formed by various arrangements of the six digits 1, 1, 1, 2, 2, 0 (note: a number can not start with zero)?

Problem 4. How many n-digit binary sequences are there without any pair of consecutive digits being the same?

Problem 5. How many different numbers can be formed by the product of two or more of the numbers 3, 4, 4, 5, 5, 6, 7, 7, 7.

Problem 6. How many ways are there for a man to invite some (nonempty) subset of his 12 fiends for a dinner?

Problem 7. How many different 9-digit numbers have at least one repeated digit (note: a number can not start with zero)?

Problem 8. Prove that for all natural values of n:

$$\sum_{k=0}^{n} \binom{n}{k} = 2^{n}.$$