Graph Theory and Combinatorics MATH-42021/52021.
Home Work 9, due on Saturday, August 18
Instructor: Prof. Artem Zvavitch
6 problems, 2pts each, YES 2 points extra!

Problem 1. *Prove that*
$$\binom{2n}{2} = 2\binom{n}{2} + n^2.$$

Problem 2. *Prove that*
$$\binom{n}{1} + 6\binom{n}{2} + 6\binom{n}{3} = n^3.$$

Problem 3. *Evaluate*
$$1^3 + 2^3 + 3^3 + \cdots + n^3.$$

Problem 4. *Evaluate*
$$\sum_{k=1}^{n} k 3^k \binom{n}{k}.$$

Problem 5. *Evaluate* $\sum\limits_{k=0}^{n} k(n-k)$.

Problem 6. *Show that*
$$\sum_{k=0}^{n} \frac{(2n)!}{(k!)^2((n-k)!)^2} = \binom{2n}{n}^2.$$