# Graph Theory and Combinatorics MATH-42021/52021. 

Home Work 4, due on Saturday, June 20
Instructor: Prof. Artem Zvavitch
10 points ( + an extra problem for 5 points)
Problem 1. Consider the graph $G$


- Does this graph contains Hamilton circuit?
- Does this graph contains Hamilton path?

Problem 2. Find a Hamiltonian path and prove that no Hamiltonian circuit exists for the following graph:


Problem 3. ( $+\mathbf{5}$ points) In chess a "knight move" consists of two squares either vertically or horizontally and then one square in a perpendicular direction. Depending on where the knight is situated, he has a minimum mobility of two moves - when in a corner- and a maximum mobility of eight moves when near the center. Let $C$ be a graph with 64 vertices corresponding to the squares of a chessboard. Let two vertices of $C$ be joint by an edge whenever a knight can go from one of the corresponding squares to the other in the move. Prove that the graph $C$ in the above problem has Hamilton path. Such a path is called "knight's tour" by puzzle enthusiasts.

Problem 4. Please, answer Yes or No to the following questions (do not forget to provide explanation)

- Is it true that if $G$ is isomorphic to $G^{\prime}$ then $\chi(G)=\chi\left(G^{\prime}\right)$ ?
- Is it true that if $\chi(G)=\chi\left(G^{\prime}\right)$, then $G$ is isomorphic to $G^{\prime}$ ?

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Problem 5. Find the chromatic number of each of the graphs


