Graph Theory and Combinatorics MATH-42021/52021.
Home Work 6, due on Saturday, June 27
Instructor: Prof. Artem Zvavitch
10 points (+an extra problem for 3 points + an extra problem for 10 points)

Problem 1. *Draw all non-isomorphic trees with six vertices.*

Problem 2. *Consider an undirected connected graph G such that the number of edges in G is less then the number of vertices, show that G is a tree.*

Problem 3. *What is the maximum number of vertices (internal and leaves) in an m-ary tree of height h?*

Problem 4. *A* **forest** *is an unconnected graph that is a disjoint union of trees. If G is an n-vertex forest of t trees, how many edges does it have?*

Problem 5. *Suppose that a chain letter is started by someone in the first week of the year. Each recipient of the chain letter mails copies on to five other people in the next week. After six weeks, him much money in postage (say 44 cents per letter) has been spent on these chain letters?*

Problem 6. (+ 3 point) *Show that the sum of the level numbers of all l leaves in a binary tree is at least $l[\log_2 l]$, and hence the average leaf level is at least $[\log_2 l]$, where $[x]$ is an integer part of x, i.e. $[4.5] = 4$ and $[6] = 6$.* **Hint: you may first show that the required sum is minimal when the tree is balanced.**

Problem 7. *Let T be an undirected tree. If the choice of vertex x to be the root yields a rooted tree of minimal height, then x is called a* **center** *of T. Show that any undirected tree has at most two centers.*

Problem 8. *Please, recover the tree from Prufer sequence $\{3, 4, 5, 5, 3, 6, 7\}$.*

Problem 9. (additional problem +10 pts) *A tree whose edge set is a subset of the edge set of the graph G is called a spanning tree of G if the tree has exactly the same vertex set as G. Does every connected graph have a spanning tree? Either give a proof or a counter-example.*