Lecture 12
MATH-42021/52021 Graph Theory and Combinatorics.

Artem Zvavitch

Department of Mathematical Sciences, Kent State University

July, 2016.

"Simple" Arragments

Artem has n candidates for a job at his computer firm in how many ways he can rank them (NO two candidates can be ranked equally!)

"Simple" Arragments

Artem has n candidates for a job at his computer firm in how many ways he can rank them (NO two candidates can be ranked equally!)

This should remind you very much of multiplication principal.

"Simple" Arragments

Artem has n candidates for a job at his computer firm in how many ways he can rank them (NO two candidates can be ranked equally!)

This should remind you very much of multiplication principal. Indeed, Artem can choose the first candidate in n ways,

Artem has n candidates for a job at his computer firm in how many ways he can rank them (NO two candidates can be ranked equally!)

This should remind you very much of multiplication principal. Indeed, Artem can choose the first candidate in n ways, FOR EACH selection he makes he may choose the next candidate in $(n-1)$ ways,

"Simple" Arragments

Artem has n candidates for a job at his computer firm in how many ways he can rank them (NO two candidates can be ranked equally!)

This should remind you very much of multiplication principal. Indeed, Artem can choose the first candidate in n ways, FOR EACH selection he makes he may choose the next candidate in $(n-1)$ ways, there are now $(n-2)$ possibilities for the third candidate and so on,

Artem has n candidates for a job at his computer firm in how many ways he can rank them (NO two candidates can be ranked equally!)

This should remind you very much of multiplication principal. Indeed, Artem can choose the first candidate in n ways, FOR EACH selection he makes he may choose the next candidate in $(n-1)$ ways, there are now $(n-2)$ possibilities for the third candidate and so on, so there are

$$n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1 = n!$$

outcomes.

"Simple" Arragments

Artem has n candidates for a job at his computer firm in how many ways he can rank them (NO two candidates can be ranked equally!)

This should remind you very much of multiplication principal. Indeed, Artem can choose the first candidate in n ways, FOR EACH selection he makes he may choose the next candidate in $(n-1)$ ways, there are now $(n-2)$ possibilities for the third candidate and so on, so there are

$$n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1 = n!$$

outcomes.

Artem has n candidates for a job at his computer firm, he must select k rank those k (NO two candidates can be ranked equally!) in how many ways he can do it?

Artem has n candidates for a job at his computer firm in how many ways he can rank them (NO two candidates can be ranked equally!)

This should remind you very much of multiplication principal. Indeed, Artem can choose the first candidate in n ways, FOR EACH selection he makes he may choose the next candidate in $(n-1)$ ways, there are now $(n-2)$ possibilities for the third candidate and so on, so there are

$$n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1 = n!$$

outcomes.

Artem has n candidates for a job at his computer firm, he must select k rank those k (NO two candidates can be ranked equally!) in how many ways he can do it?

We start in absolutely similar way as in the previous problem:

Artem has n candidates for a job at his computer firm in how many ways he can rank them (NO two candidates can be ranked equally!)

This should remind you very much of multiplication principal. Indeed, Artem can choose the first candidate in n ways, FOR EACH selection he makes he may choose the next candidate in $(n-1)$ ways, there are now $(n-2)$ possibilities for the third candidate and so on, so there are

$$n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1 = n!$$

outcomes.

Artem has n candidates for a job at his computer firm, he must select k rank those k (NO two candidates can be ranked equally!) in how many ways he can do it?

We start in absolutely similar way as in the previous problem: Artem can choose the first candidate in n ways,

Artem has n candidates for a job at his computer firm in how many ways he can rank them (NO two candidates can be ranked equally!)

This should remind you very much of multiplication principal. Indeed, Artem can choose the first candidate in n ways, FOR EACH selection he makes he may choose the next candidate in $(n-1)$ ways, there are now $(n-2)$ possibilities for the third candidate and so on, so there are

$$n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1 = n!$$

outcomes.

Artem has n candidates for a job at his computer firm, he must select k rank those k (NO two candidates can be ranked equally!) in how many ways he can do it?

We start in absolutely similar way as in the previous problem: Artem can choose the first candidate in n ways, FOR EACH selection he makes he may choose the next candidate in $(n-1)$ ways,

Artem has n candidates for a job at his computer firm in how many ways he can rank them (NO two candidates can be ranked equally!)

This should remind you very much of multiplication principal. Indeed, Artem can choose the first candidate in n ways, FOR EACH selection he makes he may choose the next candidate in $(n-1)$ ways, there are now $(n-2)$ possibilities for the third candidate and so on, so there are

$$n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1 = n!$$

outcomes.

Artem has n candidates for a job at his computer firm, he must select k rank those k (NO two candidates can be ranked equally!) in how many ways he can do it?

We start in absolutely similar way as in the previous problem: Artem can choose the first candidate in n ways, FOR EACH selection he makes he may choose the next candidate in $(n-1)$ ways, there are now $(n-2)$ possibilities for the third candidate and so on, until he have chosen k,

Artem has n candidates for a job at his computer firm in how many ways he can rank them (NO two candidates can be ranked equally!)

This should remind you very much of multiplication principal. Indeed, Artem can choose the first candidate in n ways, FOR EACH selection he makes he may choose the next candidate in $(n-1)$ ways, there are now $(n-2)$ possibilities for the third candidate and so on, so there are

$$n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1 = n!$$

outcomes.

Artem has n candidates for a job at his computer firm, he must select k rank those k (NO two candidates can be ranked equally!) in how many ways he can do it?

We start in absolutely similar way as in the previous problem: Artem can choose the first candidate in n ways, FOR EACH selection he makes he may choose the next candidate in $(n-1)$ ways, there are now $(n-2)$ possibilities for the third candidate and so on, until he have chosen k, notice that when we are choosing candidate number k, we have chosen $k-1$ of them already,

Artem has n candidates for a job at his computer firm in how many ways he can rank them (NO two candidates can be ranked equally!)

This should remind you very much of multiplication principal. Indeed, Artem can choose the first candidate in n ways, FOR EACH selection he makes he may choose the next candidate in $(n-1)$ ways, there are now $(n-2)$ possibilities for the third candidate and so on, so there are

$$n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1 = n!$$

outcomes.

Artem has n candidates for a job at his computer firm, he must select k rank those k (NO two candidates can be ranked equally!) in how many ways he can do it?

We start in absolutely similar way as in the previous problem: Artem can choose the first candidate in n ways, FOR EACH selection he makes he may choose the next candidate in $(n-1)$ ways, there are now $(n-2)$ possibilities for the third candidate and so on, until he have chosen k, notice that when we are choosing candidate number k, we have chosen $k-1$ of them already, thus we choose candidate number k out of $n-(k-1) = n-k+1$ candidates:

Artem has n candidates for a job at his computer firm in how many ways he can rank them (NO two candidates can be ranked equally!)

This should remind you very much of multiplication principal. Indeed, Artem can choose the first candidate in n ways, FOR EACH selection he makes he may choose the next candidate in $(n-1)$ ways, there are now $(n-2)$ possibilities for the third candidate and so on, so there are

$$n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1 = n!$$

outcomes.

Artem has n candidates for a job at his computer firm, he must select k rank those k (NO two candidates can be ranked equally!) in how many ways he can do it?

We start in absolutely similar way as in the previous problem: Artem can choose the first candidate in n ways, FOR EACH selection he makes he may choose the next candidate in $(n-1)$ ways, there are now $(n-2)$ possibilities for the third candidate and so on, until he have chosen k, notice that when we are choosing candidate number k, we have chosen $k-1$ of them already, thus we choose candidate number k out of $n-(k-1) = n-k+1$ candidates:

$$n \times (n-1) \times \cdots \times (n-k+2) \times (n-k+1)$$

Artem has n candidates for a job at his computer firm in how many ways he can rank them (NO two candidates can be ranked equally!)

This should remind you very much of multiplication principal. Indeed, Artem can choose the first candidate in n ways, FOR EACH selection he makes he may choose the next candidate in $(n-1)$ ways, there are now $(n-2)$ possibilities for the third candidate and so on, so there are

$$n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1 = n!$$

outcomes.

Artem has n candidates for a job at his computer firm, he must select k rank those k (NO two candidates can be ranked equally!) in how many ways he can do it?

We start in absolutely similar way as in the previous problem: Artem can choose the first candidate in n ways, FOR EACH selection he makes he may choose the next candidate in $(n-1)$ ways, there are now $(n-2)$ possibilities for the third candidate and so on, until he have chosen k, notice that when we are choosing candidate number k, we have chosen $k-1$ of them already, thus we choose candidate number k out of $n-(k-1) = n-k+1$ candidates:

$$n \times (n-1) \times \cdots \times (n-k+2) \times (n-k+1)$$

$$= \frac{n \times (n-1) \times (n-2) \times \ldots (n-k+1) \times (n-k) \times \cdots \times 2 \times 1}{(n-k) \times \cdots \times 2 \times 1} = \frac{n!}{(n-k)!}$$

outcomes.

k-permutation

A permutation of n distinct objects is an arrangement, or ordering, of the n objects.

k-permutation

A permutation of n distinct objects is an arrangement, or ordering, of the n objects.

An k-permutation of n distinct objects is an arrangement (i.e. WITH ORDER!) using k of the n objects. We will denote the number of such permutations as $P(n,k)$:

$$P(n,k) = \frac{n!}{(n-k)!}.$$

A permutation of n distinct objects is an arrangement, or ordering, of the n objects.

An k-permutation of n distinct objects is an arrangement (i.e. WITH ORDER!) using k of the n objects. We will denote the number of such permutations as $P(n,k)$:

$$P(n,k) = \frac{n!}{(n-k)!}.$$

In how many ways we can choose one person?

A permutation of n distinct objects is an arrangement, or ordering, of the n objects.

An k-permutation of n distinct objects is an arrangement (i.e. WITH ORDER!) using k of the n objects. We will denote the number of such permutations as $P(n,k)$:

$$P(n,k) = \frac{n!}{(n-k)!}.$$

In how many ways we can choose one person? Clearly, n, also

$$P(n,1) = \frac{n!}{(n-1)!} = \frac{n \times (n-1)!}{(n-1)!} = n$$

k-permutation

A permutation of n distinct objects is an arrangement, or ordering, of the n objects.

An k-permutation of n distinct objects is an arrangement (i.e. WITH ORDER!) using k of the n objects. We will denote the number of such permutations as $P(n,k)$:

$$P(n,k) = \frac{n!}{(n-k)!}.$$

In how many ways we can choose one person? Clearly, n, also

$$P(n,1) = \frac{n!}{(n-1)!} = \frac{n \times (n-1)!}{(n-1)!} = n$$

What about ranked group of two people?

A permutation of n distinct objects is an arrangement, or ordering, of the n objects.

An k-permutation of n distinct objects is an arrangement (i.e. WITH ORDER!) using k of the n objects. We will denote the number of such permutations as $P(n,k)$:

$$P(n,k) = \frac{n!}{(n-k)!}.$$

In how many ways we can choose one person? Clearly, n, also

$$P(n,1) = \frac{n!}{(n-1)!} = \frac{n \times (n-1)!}{(n-1)!} = n$$

What about ranked group of two people? $(n \times (n-1)$, indeed, the first one n options and the second is $(n-1)$ and after apply multiplication principle):

$$P(n,2) = \frac{n!}{(n-2)!} = \frac{n \times (n-1) \times (n-2)!}{(n-2)!} = n \times (n-1).$$

k-permutation

A permutation of n distinct objects is an arrangement, or ordering, of the n objects.

An k-permutation of n distinct objects is an arrangement (i.e. WITH ORDER!) using k of the n objects. We will denote the number of such permutations as $P(n,k)$:

$$P(n,k) = \frac{n!}{(n-k)!}.$$

In how many ways we can choose one person? Clearly, n, also

$$P(n,1) = \frac{n!}{(n-1)!} = \frac{n \times (n-1)!}{(n-1)!} = n$$

What about ranked group of two people? $(n \times (n-1)$, indeed, the first one n options and the second is $(n-1)$ and after apply multiplication principle):

$$P(n,2) = \frac{n!}{(n-2)!} = \frac{n \times (n-1) \times (n-2)!}{(n-2)!} = n \times (n-1).$$

Finally what about ranking all n people?

k-permutation

A permutation of n distinct objects is an arrangement, or ordering, of the n objects.

An k-permutation of n distinct objects is an arrangement (i.e. WITH ORDER!) using k of the n objects. We will denote the number of such permutations as $P(n,k)$:

$$P(n,k) = \frac{n!}{(n-k)!}.$$

In how many ways we can choose one person? Clearly, n, also

$$P(n,1) = \frac{n!}{(n-1)!} = \frac{n \times (n-1)!}{(n-1)!} = n$$

What about ranked group of two people? $(n \times (n-1)$, indeed, the first one n options and the second is $(n-1)$ and after apply multiplication principle):

$$P(n,2) = \frac{n!}{(n-2)!} = \frac{n \times (n-1) \times (n-2)!}{(n-2)!} = n \times (n-1).$$

Finally what about ranking all n people?

$$P(n,n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!.$$

NOTE WE AGREE THAT $0! = 1$.

An interesting example and k-combinations

Assume that you would like to select a soccer team of 12 player from the class of 32 people. In how many way you can do it if

An interesting example and k-combinations

Assume that you would like to select a soccer team of 12 player from the class of 32 people. In how many way you can do it if

- You will assign a number to each player?
- There is no numbers (positions) assigned?

An interesting example and k-combinations

Assume that you would like to select a soccer team of 12 player from the class of 32 people. In how many way you can do it if

- You will assign a number to each player?
- There is no numbers (positions) assigned?

The first part is exactly the same question we have solved in previous slides: $P(32,12) = \frac{32!}{(32-12)!} = \frac{32!}{20!}$.

An interesting example and k-combinations

Assume that you would like to select a soccer team of 12 player from the class of 32 people. In how many way you can do it if

- You will assign a number to each player?
- There is no numbers (positions) assigned?

The first part is exactly the same question we have solved in previous slides:
$P(32, 12) = \frac{32!}{(32-12)!} = \frac{32!}{20!}$.
What about the second part?

An interesting example and k-combinations

Assume that you would like to select a soccer team of 12 player from the class of 32 people. In how many way you can do it if

- You will assign a number to each player?
- There is no numbers (positions) assigned?

The first part is exactly the same question we have solved in previous slides:
$P(32,12) = \frac{32!}{(32-12)!} = \frac{32!}{20!}$.
What about the second part? Note that we can not simply use the $P(n,k)$ formula.

Assume that you would like to select a soccer team of 12 player from the class of 32 people. In how many way you can do it if

- You will assign a number to each player?
- There is no numbers (positions) assigned?

The first part is exactly the same question we have solved in previous slides: $P(32,12) = \frac{32!}{(32-12)!} = \frac{32!}{20!}$.
What about the second part? Note that we can not simply use the $P(n,k)$ formula. Indeed, now, when we have selected 12 people we do not car which numbers we assign to them (how we would rank them), thus all ranking become "the same" for us.

Assume that you would like to select a soccer team of 12 player from the class of 32 people. In how many way you can do it if

- You will assign a number to each player?
- There is no numbers (positions) assigned?

The first part is exactly the same question we have solved in previous slides: $P(32,12) = \frac{32!}{(32-12)!} = \frac{32!}{20!}$.
What about the second part? Note that we can not simply use the $P(n,k)$ formula. Indeed, now, when we have selected 12 people we do not car which numbers we assign to them (how we would rank them), thus all ranking become "the same" for us. Probably, this idea / "error" can actually help us!

Assume that you would like to select a soccer team of 12 player from the class of 32 people. In how many way you can do it if

- You will assign a number to each player?
- There is no numbers (positions) assigned?

The first part is exactly the same question we have solved in previous slides: $P(32,12) = \frac{32!}{(32-12)!} = \frac{32!}{20!}$.
What about the second part? Note that we can not simply use the $P(n,k)$ formula. Indeed, now, when we have selected 12 people we do not car which numbers we assign to them (how we would rank them), thus all ranking become "the same" for us. Probably, this idea / "error" can actually help us! Indeed, when we selected 12 people we can rank them $P(12,12) = 12!$ ways,

Assume that you would like to select a soccer team of 12 player from the class of 32 people. In how many way you can do it if

- You will assign a number to each player?
- There is no numbers (positions) assigned?

The first part is exactly the same question we have solved in previous slides:
$P(32,12) = \frac{32!}{(32-12)!} = \frac{32!}{20!}$.
What about the second part? Note that we can not simply use the $P(n,k)$ formula. Indeed, now, when we have selected 12 people we do not car which numbers we assign to them (how we would rank them), thus all ranking become "the same" for us. Probably, this idea / "error" can actually help us! Indeed, when we selected 12 people we can rank them $P(12,12) = 12!$ ways, so we each time "over-calculated" 12! options

Assume that you would like to select a soccer team of 12 player from the class of 32 people. In how many way you can do it if

- You will assign a number to each player?
- There is no numbers (positions) assigned?

The first part is exactly the same question we have solved in previous slides: $P(32,12) = \frac{32!}{(32-12)!} = \frac{32!}{20!}$.
What about the second part? Note that we can not simply use the $P(n,k)$ formula. Indeed, now, when we have selected 12 people we do not car which numbers we assign to them (how we would rank them), thus all ranking become "the same" for us. Probably, this idea / "error" can actually help us! Indeed, when we selected 12 people we can rank them $P(12,12) = 12!$ ways, so we each time "over-calculated" 12! options so the answer is

$$\frac{P(32,12)}{P(12,12)} = \frac{32!}{(32-12)! \times 12!}$$

Assume that you would like to select a soccer team of 12 player from the class of 32 people. In how many way you can do it if

- You will assign a number to each player?
- There is no numbers (positions) assigned?

The first part is exactly the same question we have solved in previous slides:
$P(32,12) = \frac{32!}{(32-12)!} = \frac{32!}{20!}$.
What about the second part? Note that we can not simply use the $P(n,k)$ formula. Indeed, now, when we have selected 12 people we do not car which numbers we assign to them (how we would rank them), thus all ranking become "the same" for us. Probably, this idea / "error" can actually help us! Indeed, when we selected 12 people we can rank them $P(12,12) = 12!$ ways, so we each time "over-calculated" 12! options so the answer is

$$\frac{P(32,12)}{P(12,12)} = \frac{32!}{(32-12)! \times 12!}$$

k-combination

An k-combination of n distinct objects is an unordered selection or subset of k out of the n objects. We will denote the number of such selection as $C(n,k)$:

$$C(n,k) = \binom{n}{k} = \frac{P(n,k)}{P(k,k)} = \frac{n!}{k!(n-k)!}.$$

Why the name "Binomial coefficients?"

k-combination / Binomial Coefficients

An k-combination of n distinct objects is an unordered selection or subset of k out of the n objects. We will denote the number of such selection as $C(n,k)$:

$$C(n,k) = \binom{n}{k} = \frac{P(n,k)}{P(k,k)} = \frac{n!}{k!(n-k)!}.$$

Why the name "Binomial coefficients?"

k-combination / Binomial Coefficients

An k-combination of n distinct objects is an unordered selection or subset of k out of the n objects. We will denote the number of such selection as $C(n,k)$:

$$C(n,k) = \binom{n}{k} = \frac{P(n,k)}{P(k,k)} = \frac{n!}{k!(n-k)!}.$$

Let us compute $(x+y)^n$ we know that

Why the name "Binomial coefficients?"

k-combination / Binomial Coefficients

An k-combination of n distinct objects is an unordered selection or subset of k out of the n objects. We will denote the number of such selection as $C(n,k)$:

$$C(n,k) = \binom{n}{k} = \frac{P(n,k)}{P(k,k)} = \frac{n!}{k!(n-k)!}.$$

Let us compute $(x+y)^n$ we know that

$$(x+y)^n = \underbrace{(x+y)\times(x+y)\times(x+y)\times\cdots\times(x+y)}_{n-\text{times}}$$

k-combination / Binomial Coefficients

An k-combination of n distinct objects is an unordered selection or subset of k out of the n objects. We will denote the number of such selection as $C(n,k)$:

$$C(n,k) = \binom{n}{k} = \frac{P(n,k)}{P(k,k)} = \frac{n!}{k!(n-k)!}.$$

Let us compute $(x+y)^n$ we know that

$$(x+y)^n = \underbrace{(x+y)\times(x+y)\times(x+y)\times\cdots\times(x+y)}_{n-\text{times}}$$

Now we need to multiply, we need to pick ONE element from each of multipliers $(x+y)$ it can be x or y

Why the name "Binomial coefficients?"

k-combination / Binomial Coefficients

An k-combination of n distinct objects is an unordered selection or subset of k out of the n objects. We will denote the number of such selection as $C(n,k)$:

$$C(n,k) = \binom{n}{k} = \frac{P(n,k)}{P(k,k)} = \frac{n!}{k!(n-k)!}.$$

Let us compute $(x+y)^n$ we know that

$$(x+y)^n = \underbrace{(x+y)\times(x+y)\times(x+y)\times\cdots\times(x+y)}_{n-\text{times}}$$

Now we need to multiply, we need to pick ONE element from each of multipliers $(x+y)$ it can be x or y if we pick k times - x,

Why the name "Binomial coefficients?"

k-combination / Binomial Coefficients

An k-combination of n distinct objects is an unordered selection or subset of k out of the n objects. We will denote the number of such selection as $C(n,k)$:

$$C(n,k) = \binom{n}{k} = \frac{P(n,k)}{P(k,k)} = \frac{n!}{k!(n-k)!}.$$

Let us compute $(x+y)^n$ we know that

$$(x+y)^n = \underbrace{(x+y) \times (x+y) \times (x+y) \times \cdots \times (x+y)}_{n-\text{times}}$$

Now we need to multiply, we need to pick ONE element from each of multipliers $(x+y)$ it can be x or y if we pick k times - x, then we must pick $(n-k)$ - y, and the outcome is $x^k y^{n-k}$.

Why the name "Binomial coefficients?"

k-combination / Binomial Coefficients

An k-combination of n distinct objects is an unordered selection or subset of k out of the n objects. We will denote the number of such selection as $C(n,k)$:

$$C(n,k) = \binom{n}{k} = \frac{P(n,k)}{P(k,k)} = \frac{n!}{k!(n-k)!}.$$

Let us compute $(x+y)^n$ we know that

$$(x+y)^n = \underbrace{(x+y)\times(x+y)\times(x+y)\times\cdots\times(x+y)}_{n-\text{times}}$$

Now we need to multiply, we need to pick ONE element from each of multipliers $(x+y)$ it can be x or y if we pick k times - x, then we must pick $(n-k)$ - y, and the outcome is $x^k y^{n-k}$. But in how many ways we can pick a place from which we pick k of x's?????

Why the name "Binomial coefficients?"

k-combination / Binomial Coefficients

An k-combination of n distinct objects is an unordered selection or subset of k out of the n objects. We will denote the number of such selection as $C(n,k)$:

$$C(n,k) = \binom{n}{k} = \frac{P(n,k)}{P(k,k)} = \frac{n!}{k!(n-k)!}.$$

Let us compute $(x+y)^n$ we know that

$$(x+y)^n = \underbrace{(x+y)\times(x+y)\times(x+y)\times\cdots\times(x+y)}_{n-\text{times}}$$

Now we need to multiply, we need to pick ONE element from each of multipliers $(x+y)$ it can be x or y if we pick k times - x, then we must pick $(n-k)$ - y, and the outcome is $x^k y^{n-k}$. But in how many ways we can pick a place from which we pick k of x's????? Note, we do note care about the order! Thus exactly from

$$\binom{n}{k}$$

places!!

Why the name "Binomial coefficients?"

k-combination / Binomial Coefficients

An k-combination of n distinct objects is an unordered selection or subset of k out of the n objects. We will denote the number of such selection as $C(n,k)$:

$$C(n,k) = \binom{n}{k} = \frac{P(n,k)}{P(k,k)} = \frac{n!}{k!(n-k)!}.$$

Let us compute $(x+y)^n$ we know that

$$(x+y)^n = \underbrace{(x+y)\times(x+y)\times(x+y)\times\cdots\times(x+y)}_{n-\text{times}}$$

Now we need to multiply, we need to pick ONE element from each of multipliers $(x+y)$ it can be x or y if we pick k times - x, then we must pick $(n-k)$ - y, and the outcome is $x^k y^{n-k}$. But in how many ways we can pick a place from which we pick k of x's????? Note, we do note care about the order! Thus exactly from

$$\binom{n}{k}$$

places!! And the final formula is

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.$$

Examples!

How many ways there are to arrange 5 letters in the word Artem?

Examples!

How many ways there are to arrange 5 letters in the word Artem?

Easy (now) question, there are 5 letters, any arrangement will create a different sequence so $P(5,5) = 5!$.

Examples!

How many ways there are to arrange 5 letters in the word Artem?

Easy (now) question, there are 5 letters, any arrangement will create a different sequence so $P(5,5) = 5!$.

How many ways there are to arrange 7 letters in the word systems?

Examples!

How many ways there are to arrange 5 letters in the word Artem?

Easy (now) question, there are 5 letters, any arrangement will create a different sequence so $P(5,5) = 5!$.

How many ways there are to arrange 7 letters in the word systems?

Not so easy (for now) question, there are 7 letters,

Examples!

How many ways there are to arrange 5 letters in the word Artem?

Easy (now) question, there are 5 letters, any arrangement will create a different sequence so $P(5,5) = 5!$.

How many ways there are to arrange 7 letters in the word systems?

Not so easy (for now) question, there are 7 letters, but there is a problem 3 of them are the same so if we would answer $P(7,7)$ we will over-calculate.

Examples!

How many ways there are to arrange 5 letters in the word Artem?

Easy (now) question, there are 5 letters, any arrangement will create a different sequence so $P(5,5) = 5!$.

How many ways there are to arrange 7 letters in the word systems?

Not so easy (for now) question, there are 7 letters, but there is a problem 3 of them are the same so if we would answer $P(7,7)$ we will over-calculate. But can we say by how much?

Examples!

How many ways there are to arrange 5 letters in the word Artem?

Easy (now) question, there are 5 letters, any arrangement will create a different sequence so $P(5,5) = 5!$.

How many ways there are to arrange 7 letters in the word systems?

Not so easy (for now) question, there are 7 letters, but there is a problem 3 of them are the same so if we would answer $P(7,7)$ we will over-calculate. But can we say by how much? EACH TIME by $P(3,3)$

Examples!

How many ways there are to arrange 5 letters in the word Artem?

Easy (now) question, there are 5 letters, any arrangement will create a different sequence so $P(5,5) = 5!$.

How many ways there are to arrange 7 letters in the word systems?

Not so easy (for now) question, there are 7 letters, but there is a problem 3 of them are the same so if we would answer $P(7,7)$ we will over-calculate. But can we say by how much? EACH TIME by $P(3,3)$ so the answer is

$$\frac{P(7,7)}{P(3,3)} = \frac{7!}{3!}$$

Examples!

How many ways there are to arrange 5 letters in the word Artem?

Easy (now) question, there are 5 letters, any arrangement will create a different sequence so $P(5,5) = 5!$.

How many ways there are to arrange 7 letters in the word systems?

Not so easy (for now) question, there are 7 letters, but there is a problem 3 of them are the same so if we would answer $P(7,7)$ we will over-calculate. But can we say by how much? EACH TIME by $P(3,3)$ so the answer is

$$\frac{P(7,7)}{P(3,3)} = \frac{7!}{3!}$$

There is another way to see it.

Examples!

How many ways there are to arrange 5 letters in the word Artem?

Easy (now) question, there are 5 letters, any arrangement will create a different sequence so $P(5,5) = 5!$.

How many ways there are to arrange 7 letters in the word systems?

Not so easy (for now) question, there are 7 letters, but there is a problem 3 of them are the same so if we would answer $P(7,7)$ we will over-calculate. But can we say by how much? EACH TIME by $P(3,3)$ so the answer is

$$\frac{P(7,7)}{P(3,3)} = \frac{7!}{3!}$$

There is another way to see it. Forget about problematic letters for a moment, start with y, there are 7 positions for y,

Examples!

How many ways there are to arrange 5 letters in the word Artem?

Easy (now) question, there are 5 letters, any arrangement will create a different sequence so $P(5,5) = 5!$.

How many ways there are to arrange 7 letters in the word systems?

Not so easy (for now) question, there are 7 letters, but there is a problem 3 of them are the same so if we would answer $P(7,7)$ we will over-calculate. But can we say by how much? EACH TIME by $P(3,3)$ so the answer is

$$\frac{P(7,7)}{P(3,3)} = \frac{7!}{3!}$$

There is another way to see it. Forget about problematic letters for a moment, start with y, there are 7 positions for y, next t - there are 6 positions,

Examples!

How many ways there are to arrange 5 letters in the word Artem?

Easy (now) question, there are 5 letters, any arrangement will create a different sequence so $P(5,5) = 5!$.

How many ways there are to arrange 7 letters in the word systems?

Not so easy (for now) question, there are 7 letters, but there is a problem 3 of them are the same so if we would answer $P(7,7)$ we will over-calculate. But can we say by how much? EACH TIME by $P(3,3)$ so the answer is

$$\frac{P(7,7)}{P(3,3)} = \frac{7!}{3!}$$

There is another way to see it. Forget about problematic letters for a moment, start with y, there are 7 positions for y, next t - there are 6 positions, and 5 for e, and we can put s on places which were left open.

How many ways there are to arrange 5 letters in the word Artem?

Easy (now) question, there are 5 letters, any arrangement will create a different sequence so $P(5,5) = 5!$.

How many ways there are to arrange 7 letters in the word systems?

Not so easy (for now) question, there are 7 letters, but there is a problem 3 of them are the same so if we would answer $P(7,7)$ we will over-calculate. But can we say by how much? EACH TIME by $P(3,3)$ so the answer is

$$\frac{P(7,7)}{P(3,3)} = \frac{7!}{3!}$$

There is another way to see it. Forget about problematic letters for a moment, start with y, there are 7 positions for y, next t - there are 6 positions, and 5 for e, finally there are 4 places for m and we can put s on places which were left open. Using multiplicative principle we get that the answer is

$$7 \times 6 \times 5 \times 4 = \frac{7!}{3!}$$

Very VERY Famous Example

How many ways there are to arrange 11 letters in the word MISSISSIPPI?

Very VERY Famous Example

How many ways there are to arrange 11 letters in the word MISSISSIPPI?

There is nothing to be scared about this famous problem!

Very VERY Famous Example

How many ways there are to arrange 11 letters in the word MISSISSIPPI?

There is nothing to be scared about this famous problem! There are 11 letters, but, yes, there is a problem: 4 of them are repeated "I", 4 of them repetitions of S and there are 2 repetitions of P so if we would answer $P(11, 11)$ we will over-calculate (much over-calculate).

How many ways there are to arrange 11 letters in the word MISSISSIPPI?

There is nothing to be scared about this famous problem! There are 11 letters, but, yes, there is a problem: 4 of them are repeated "I", 4 of them repetitions of S and there are 2 repetitions of P so if we would answer $P(11, 11)$ we will over-calculate (much over-calculate).
But, again, can we say by how much? EACH TIME by $P(4, 4)$ because we have forgotten that there are 4 - "I",

Very VERY Famous Example

How many ways there are to arrange 11 letters in the word MISSISSIPPI?

There is nothing to be scared about this famous problem! There are 11 letters, but, yes, there is a problem: 4 of them are repeated "I", 4 of them repetitions of S and there are 2 repetitions of P so if we would answer $P(11, 11)$ we will over-calculate (much over-calculate).
But, again, can we say by how much? EACH TIME by $P(4, 4)$ because we have forgotten that there are 4 - "I", moreover, even if we correct it, we now "forgotten" about 4 of them repetitions of S (and must correct it by another $P(4, 4)$),

Very VERY Famous Example

How many ways there are to arrange 11 letters in the word MISSISSIPPI?

There is nothing to be scared about this famous problem! There are 11 letters, but, yes, there is a problem: 4 of them are repeated "I", 4 of them repetitions of S and there are 2 repetitions of P so if we would answer $P(11, 11)$ we will over-calculate (much over-calculate).
But, again, can we say by how much? EACH TIME by $P(4, 4)$ because we have forgotten that there are 4 - "I", moreover, even if we correct it, we now "forgotten" about 4 of them repetitions of S (and must correct it by another $P(4, 4)$), finally we must correct by $P(2, 2)$ to offset repetitions of p.

Very VERY Famous Example

How many ways there are to arrange 11 letters in the word MISSISSIPPI?

There is nothing to be scared about this famous problem! There are 11 letters, but, yes, there is a problem: 4 of them are repeated "I", 4 of them repetitions of S and there are 2 repetitions of P so if we would answer $P(11,11)$ we will over-calculate (much over-calculate).
But, again, can we say by how much? EACH TIME by $P(4,4)$ because we have forgotten that there are 4 - "I", moreover, even if we correct it, we now "forgotten" about 4 of them repetitions of S (and must correct it by another $P(4,4)$), finally we must correct by $P(2,2)$ to offset repetitions of p. Thus the final formula is

$$\frac{P(11,11)}{P(4,4)\times P(4,4)\times P(2,2)}$$