Lecture 14
MATH-42021/52021 Graph Theory and Combinatorics.

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This is a process of distribution of r **distinct** objects into n **different** boxes. To compute the number of outcomes we put distinct objects in a row, and give each of them one of n different box names, thus the number of outcomes is

$$n \times n \times n \times \cdots \times n = n^r.$$

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$$P(100;20,20,20,20,20)=\binom{100}{20}\binom{80}{20}\binom{60}{20}\binom{40}{20}\binom{20}{20}=\frac{100!}{20!20!20!20!20!}.$$

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This is a process of distribution of r **distinct** objects into n **different** boxes, WITH requirement that r_i object must go in box $1,2,\dots,n$ (and $r=r_1+r_2+\cdots+r_n$). The number of outcomes is

$$P(r;r_1,r_2,r_3,\dots,r_n)=\binom{r}{r_1}\binom{r-r_1}{r_2}\binom{r-r_1-r_2}{r_3}\cdots\binom{r-r_1-r_2-\cdots-r_{n-1}}{r_n}=\frac{r!}{r_1!r_2!r_3!\cdots r_n!}.$$

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This is a process of distribution of r **identical** objects into n **different** boxes, is equivalent to choosing an (unordered) subset of r box names with repetition from among the n choices of boxes. Thus there are

$$\binom{r+n-1}{r} = \frac{(r+n-1)!}{r!(n-1)!}.$$

distributions of the r identical objects.

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