Lecture 14
MATH-42021/52021 Graph Theory and Combinatorics.

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This is a process of distribution of r **distinct** objects into n **different** boxes. To compute the number of outcomes we put distinct objects in a row, and give each of them one of n different box names, thus the number of outcomes is

$$n \times n \times n \times \cdots \times n = n^r.$$

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Solution: Note that now we do not have so much freedom, we can not simply randomly give diplomats numbers for the countries (we must send 20 of them to each country!). Thus we need to use our formula for the number of arrangements:

$$P(100; 20, 20, 20, 20, 20) = \binom{100}{20}\binom{80}{20}\binom{60}{20}\binom{40}{20}\binom{20}{20} = \frac{100!}{20!20!20!20!20!}.$$

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This is a process of distribution of r **distinct** objects into n **different** boxes, WITH requirement that r_i object must go in box $1, 2, \dots, n$ (and $r = r_1 + r_2 + \cdots + r_n$). The number of outcomes is

$$P(r; r_1, r_2, r_3, \dots, r_n) = \binom{r}{r_1}\binom{r - r_1}{r_2}\binom{r - r_1 - r_2}{r_3}\cdots\binom{r - r_1 - r_2 - \cdots - r_{n-1}}{r_n} = \frac{r!}{r_1!r_2!r_3!\cdots r_n!}.$$

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This is a process of distribution of r **identical** objects into n **different** boxes, is equivalent to choosing an (unordered) subset of r box names with repetition from among the n choices of boxes. Thus there are

$$\binom{r+n-1}{r} = \frac{(r+n-1)!}{r!(n-1)!}.$$

distributions of the r identical objects.

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