Lecture 17 MATH-42021/52021 Graph Theory and Combinatorics.

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Consider a sequence of numbers $a_0, a_1, a_2, ..., a_n$. The generating function g(x) is a polynomial defined as

$$g(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$
.

Note that we also can consider an infinite sequence of numbers $a_0, a_1, a_2, ..., a_n, ...$ Then the generating function g(x) is a series which is defined as

$$g(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n + \dots$$

 $0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots$

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But the generating function of the first sequence is xG(x)

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But the generating function of the first sequence is xG(x) and the generating function of the second is $x^2G(x)$ and the third is just x.

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$$G(x) = x^2 G(x) + x G(x) + x$$

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$$\frac{x}{(x+\frac{1+\sqrt{5}}{2})(x+\frac{1-\sqrt{5}}{2})} = \frac{A}{x+\frac{1+\sqrt{5}}{2}} + \frac{B}{x+\frac{1-\sqrt{5}}{2}}$$

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Next you simply the right-hand side (just add up the fraction) compare the numerator to the numerator of the left-hand side and find A and B.

Fibonacci numbers are the numbers in the following integer sequence, called the Fibonacci sequence: $a_0 = 0$, $a_1 = 1$ and $a_n = a_{n-1} + a_{n-2}$.

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so

$$a_{\text{II}} = \frac{1}{\sqrt{5}} \left[\left(\frac{2}{\sqrt{5}-1} \right)^{\text{II}} - \left(\frac{-2}{\sqrt{5}+1} \right)^{\text{II}} \right]$$

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Solution: Let l_1 -be the number of green ball, we select; l_2 -white; l_3 -blue; l_4 -red. Then

 $l_1 + l_2 + l_3 + l_4 = n$ and $l_1, l_2, l_3, l_4 \in \{0, 1, 2, 3\}$.

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Thus a_n is simply the coefficient of $x^{l_1+l_2+l_3+l_4} = x^n$ in $(1+x+x^2+x^3)^4$ and the generating function we are looking for is

$$G(x) = (1 + x + x^2 + x^3)^4.$$

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and

 $l_1 \in \{0, 1, 2, 3\}; l_2 \in \{0, 2, 4\}; l_3 \in \{0, 1, 2, 3, 4, 5\}; l_4 \in \{0, 1, 2\}; l_3 \in \{1, 3, 5, 7\}.$

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$$l_1 + l_2 + l_3 + l_4 + l_5 = n$$

and

$$I_1 \in \{0,1,2,3\}; I_2 \in \{0,2,4\}; I_3 \in \{0,1,2,3,4,5\}; I_4 \in \{0,1,2\}; I_3 \in \{1,3,5,7\}.$$

Thus the generating function is

$$G(x) = (1 + x + x^{2} + x^{3})(1 + x^{2} + x^{4})(1 + x + x^{2} + x^{3} + x^{4} + x^{5})(1 + x + x^{2})(x + x^{3} + x^{5} + x^{7}).$$

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Solution: We know that

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$$a_n = \binom{n+1}{1}$$

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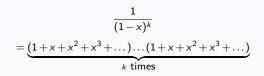
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