Lecture 1.2, MATH-42021/52021 Graph Theory and Combinatorics.

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Department of Mathematical Sciences, Kent State University

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Isomorphism

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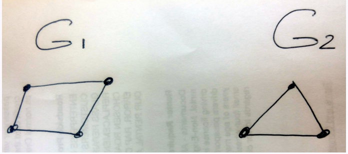

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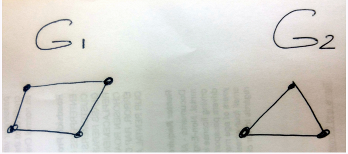

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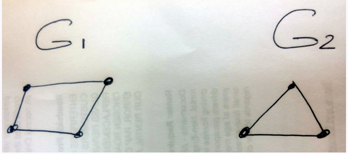

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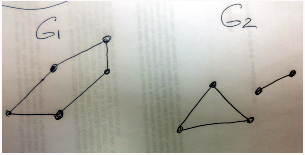

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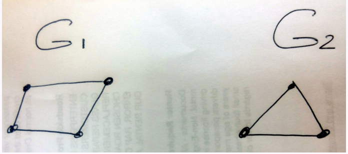

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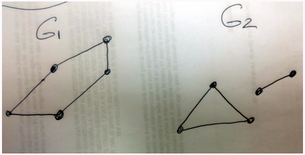

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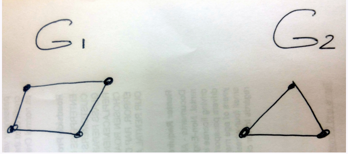

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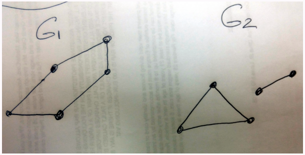

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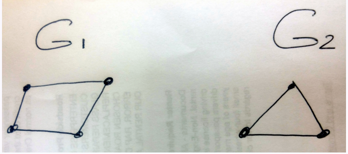

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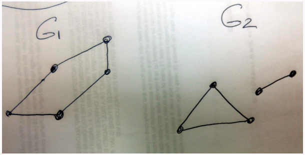

The number of vertices is the same, but still, they are very different. Number of edges is not the same. One is connected another not!

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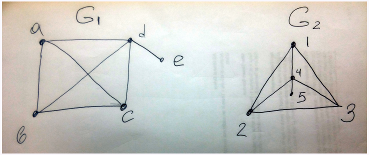

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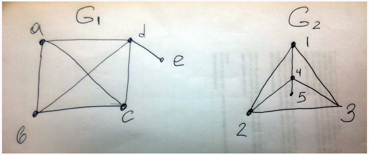

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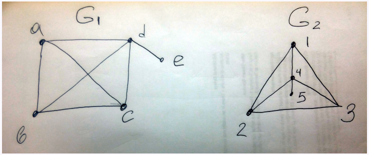

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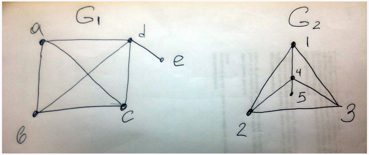

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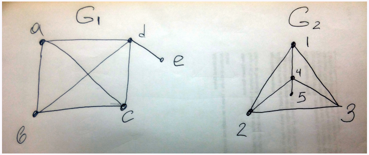

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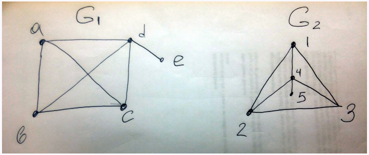

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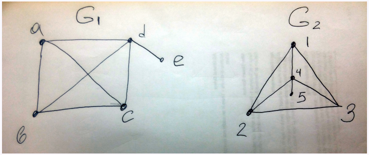

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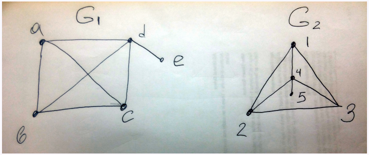

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Next it is not hard to check that f is isomorphism, indeed this is one to one map and we can check the property of preserving the adjacency (for example f maps (e, d) to $(5, 4)$).

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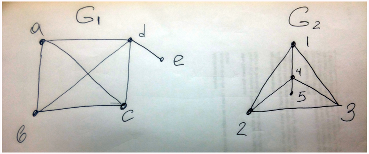

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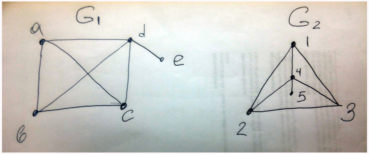

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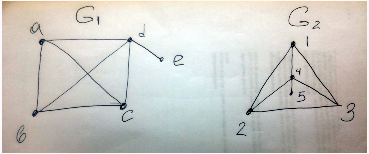

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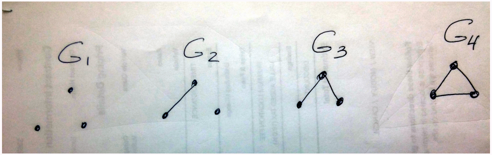

Subgraphs

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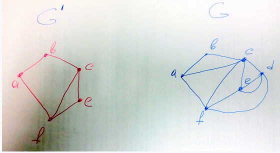

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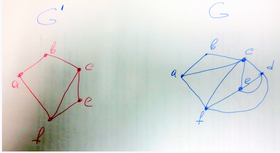

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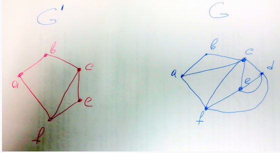

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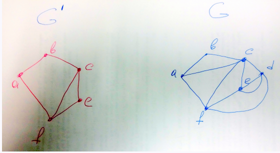

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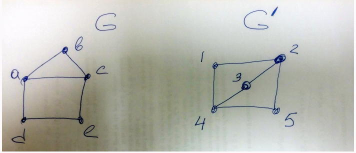

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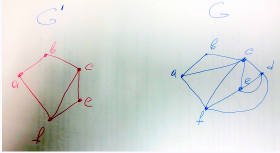

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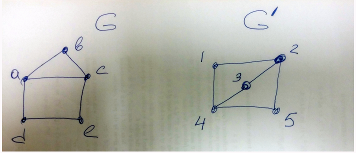

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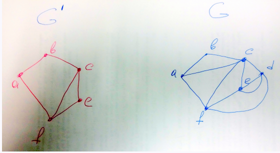

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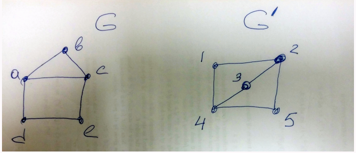

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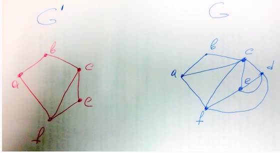

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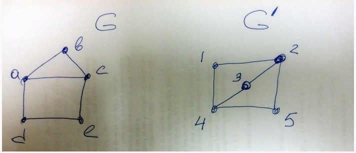

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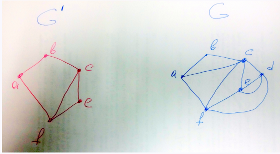

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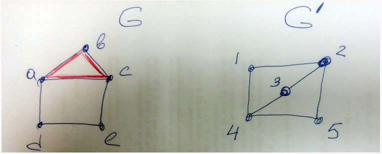

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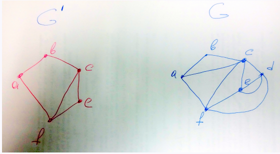

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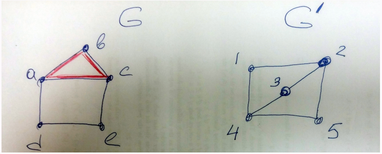

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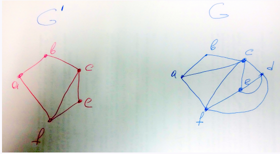
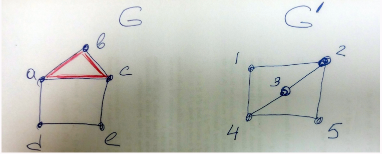
Subgraphs

A subgraph is a graph formed by a subset of vertices and edges of a larger graph.



Subgraphs are super useful when we study isomorphisms. Remember, the isomorphisms must preserve "structure" thus must preserve subgraphs. So do you think those two graphs are isomorphic:



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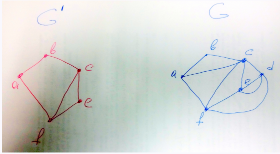

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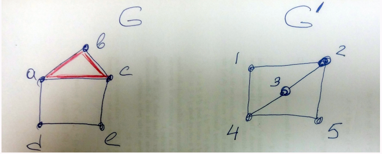

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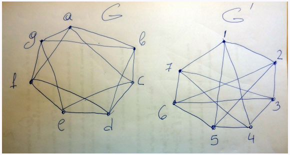
NOTICE: this is an example of two graphs, with the same, vertex, edge and degree count which are NOT isomorphic.

Another example

Are those two graphs isomorphic?



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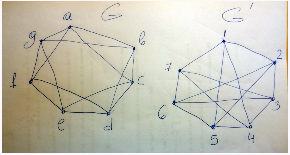

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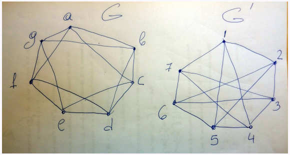

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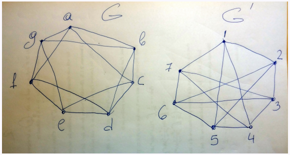

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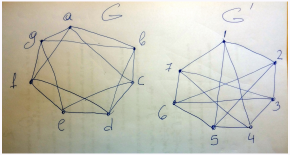

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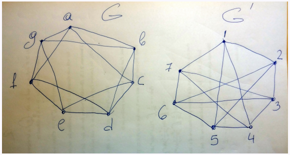

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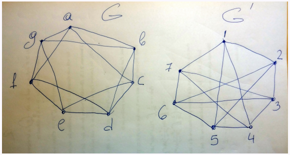

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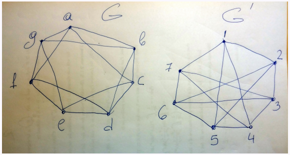

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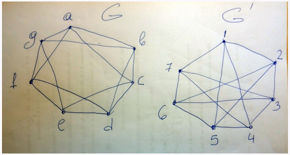

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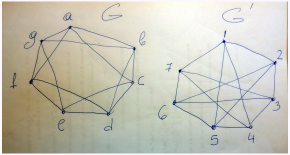

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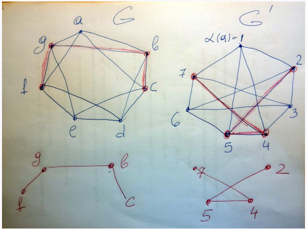

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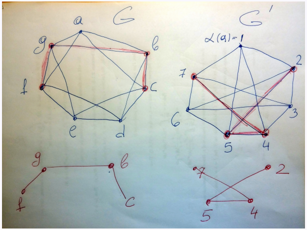

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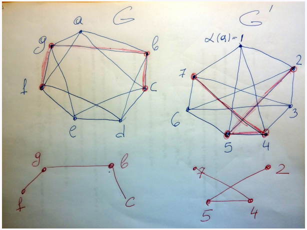

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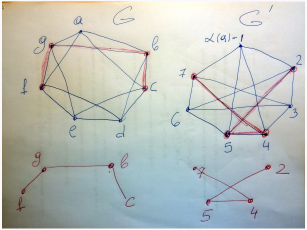

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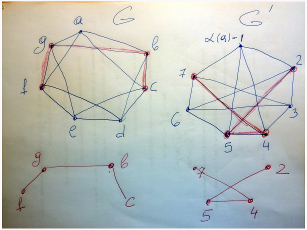

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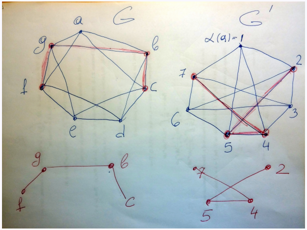

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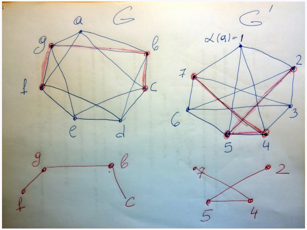

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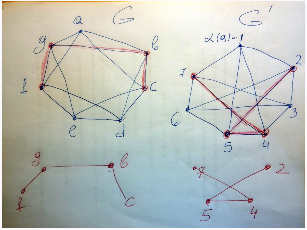

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Yet, another example + a cool idea

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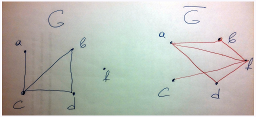

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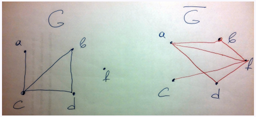

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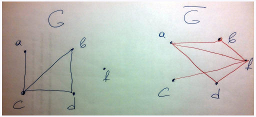

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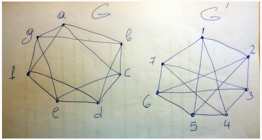

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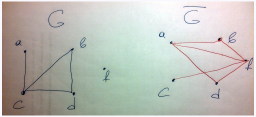

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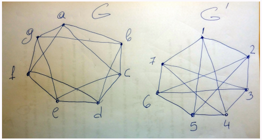

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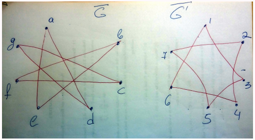

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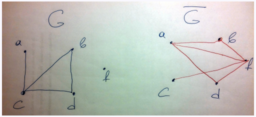

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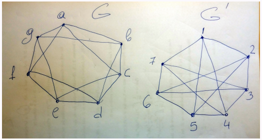

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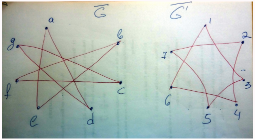

Both of them are circuits of length 7, thus isomorphic, and solution became almost trivial with this cool trick!!!