

Lecture 1.2, MATH-42021/52021 Graph Theory and Combinatorics.

Artem Zvavitch

Department of Mathematical Sciences, Kent State University

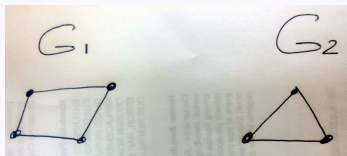
July, 2018.

The word isomorphism comes from Ancient Greek: isos – "equal", and morphe — "form" or "shape"). So it is used to describe similarity of different objects.

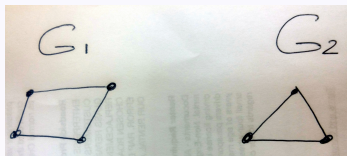
The word isomorphism comes from Ancient Greek: isos – "equal", and morphe — "form" or "shape"). So it is used to describe similarity of different objects. We would like to understand and define what is it for graphs to be "the same".

Isomorphism

The word isomorphism comes from Ancient Greek: isos – "equal", and morphe — "form" or "shape"). So it is used to describe similarity of different objects. We would like to understand and define what is it for graphs to be "the same". What do you think about those graphs:

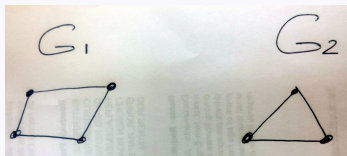


The word isomorphism comes from Ancient Greek: isos – "equal", and morphe — "form" or "shape"). So it is used to describe similarity of different objects. We would like to understand and define what is it for graphs to be "the same". What do you think about those graphs:

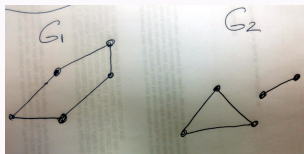


They are different -> different number of vertices.

The word isomorphism comes from Ancient Greek: isos – "equal", and morphe — "form" or "shape"). So it is used to describe similarity of different objects. We would like to understand and define what is it for graphs to be "the same". What do you think about those graphs:

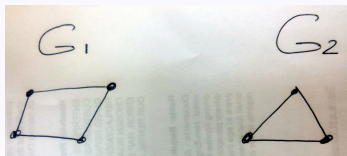


They are different \rightarrow different number of vertices. What do you think about those graphs:

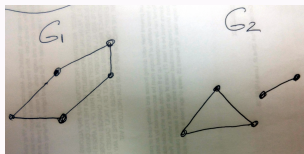


Isomorphism

The word isomorphism comes from Ancient Greek: isos – "equal", and morphe — "form" or "shape"). So it is used to describe similarity of different objects. We would like to understand and define what is it for graphs to be "the same". What do you think about those graphs:

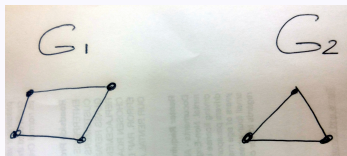


They are different \rightarrow different number of vertices. What do you think about those graphs:

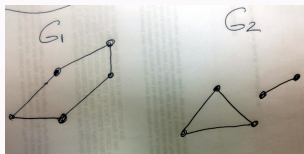


The number of vertices is the same,

The word isomorphism comes from Ancient Greek: isos – "equal", and morphe — "form" or "shape"). So it is used to describe similarity of different objects. We would like to understand and define what is it for graphs to be "the same". What do you think about those graphs:

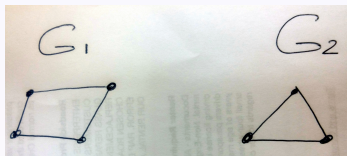


They are different \rightarrow different number of vertices. What do you think about those graphs:

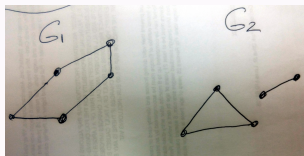


The number of vertices is the same, but still, they are very different.

The word isomorphism comes from Ancient Greek: isos – "equal", and morphe — "form" or "shape"). So it is used to describe similarity of different objects. We would like to understand and define what is it for graphs to be "the same". What do you think about those graphs:



They are different \rightarrow different number of vertices. What do you think about those graphs:

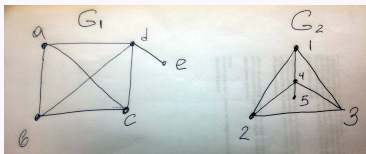


The number of vertices is the same, but still, they are very different. Number of edges is not the same. One is connected another not!

Isomorphism

The word isomorphism comes from Ancient Greek: isos – "equal", and morphe — "form" or "shape"). So it is used to describe similarity of different objects.

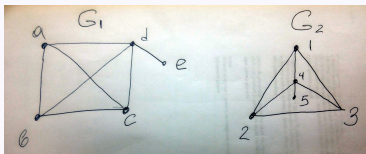
We would like to understand and define what is it for graphs to be "the same". What do you think about those graphs:



Isomorphism

The word isomorphism comes from Ancient Greek: isos – "equal", and morphe — "form" or "shape"). So it is used to describe similarity of different objects.

We would like to understand and define what is it for graphs to be "the same". What do you think about those graphs:

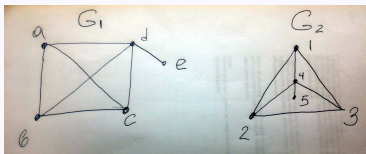


Number of vertices the same, number of edges the same (NOTE THIS IS NOT ENOUGH TO BE THE SAME).

Isomorphism

The word isomorphism comes from Ancient Greek: isos – "equal", and morphe — "form" or "shape"). So it is used to describe similarity of different objects.

We would like to understand and define what is it for graphs to be "the same". What do you think about those graphs:

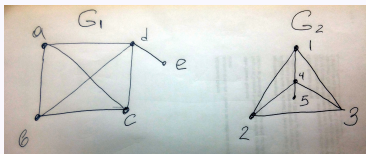


Number of vertices the same, number of edges the same (NOTE THIS IS NOT ENOUGH TO BE THE SAME). Probably, there is a hope

Isomorphism

The word isomorphism comes from Ancient Greek: isos – "equal", and morphe — "form" or "shape"). So it is used to describe similarity of different objects.

We would like to understand and define what is it for graphs to be "the same". What do you think about those graphs:

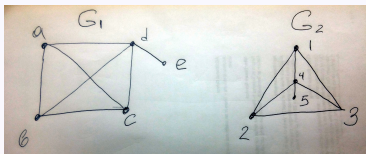


Number of vertices the same, number of edges the same (NOTE THIS IS NOT ENOUGH TO BE THE SAME). Probably, there is a hope and indeed we can pull vertex 5 "down" in graph G_2 (together with vertex 4) so that 4 and 5 goes out of the triangle 1, 2, 3, and the graph looks the same.

Isomorphism

The word isomorphism comes from Ancient Greek: isos – "equal", and morphe — "form" or "shape"). So it is used to describe similarity of different objects.

We would like to understand and define what is it for graphs to be "the same". What do you think about those graphs:



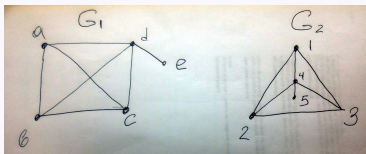
Number of vertices the same, number of edges the same (NOTE THIS IS NOT ENOUGH TO BE THE SAME). Probably, there is a hope and indeed we can pull vertex 5 "down" in graph G_2 (together with vertex 4) so that 4 and 5 goes out of the triangle 1, 2, 3, and the graph looks the same. Now we must give a precise definition

Two graphs G_1 and G_2 are called **isomorphic** if there exists a one-to-one correspondence between the vertices in G_1 and the vertices in G_2 such that a pair of vertices are adjacent in G_1 if and only if the corresponding pair of vertices are adjacent in G_2

Isomorphism

The word isomorphism comes from Ancient Greek: isos – "equal", and morphe — "form" or "shape"). So it is used to describe similarity of different objects.

We would like to understand and define what is it for graphs to be "the same". What do you think about those graphs:



Number of vertices the same, number of edges the same (NOTE THIS IS NOT ENOUGH TO BE THE SAME). Probably, there is a hope and indeed we can pull vertex 5 "down" in graph G_2 (together with vertex 4) so that 4 and 5 goes out of the triangle 1, 2, 3, and the graph looks the same. Now we must give a precise definition

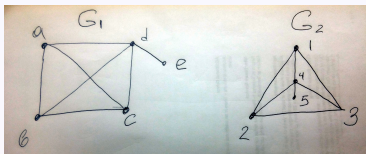
Two graphs G_1 and G_2 are called **isomorphic** if there exists a one-to-one correspondence between the vertices in G_1 and the vertices in G_2 such that a pair of vertices are adjacent in G_1 if and only if the corresponding pair of vertices are adjacent in G_2

Now we can make "pull vertex 5" story more mathematical:

Isomorphism

The word isomorphism comes from Ancient Greek: isos – "equal", and morphé — "form" or "shape"). So it is used to describe similarity of different objects.

We would like to understand and define what is it for graphs to be "the same". What do you think about those graphs:



Number of vertices the same, number of edges the same (NOTE THIS IS NOT ENOUGH TO BE THE SAME). Probably, there is a hope and indeed we can pull vertex 5 "down" in graph G_2 (together with vertex 4) so that 4 and 5 goes out of the triangle 1, 2, 3, and the graph looks the same. Now we must give a precise definition

Two graphs G_1 and G_2 are called **isomorphic** if there exists a one-to-one correspondence between the vertices in G_1 and the vertices in G_2 such that a pair of vertices are adjacent in G_1 if and only if the corresponding pair of vertices are adjacent in G_2

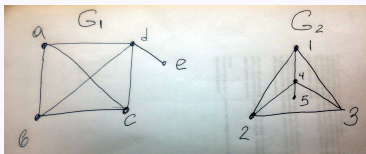
Now we can make "pull vertex 5" story more mathematical: let f be the map between vertices of G_1 and G_2 defined in the following way:

$$f(e) = 5; f(d) = 4, f(b) = 1, f(c) = 2, f(a) = 3.$$

Isomorphism

The word isomorphism comes from Ancient Greek: isos – "equal", and morphe — "form" or "shape"). So it is used to describe similarity of different objects.

We would like to understand and define what is it for graphs to be "the same". What do you think about those graphs:



Number of vertices the same, number of edges the same (NOTE THIS IS NOT ENOUGH TO BE THE SAME). Probably, there is a hope and indeed we can pull vertex 5 "down" in graph G_2 (together with vertex 4) so that 4 and 5 goes out of the triangle 1, 2, 3, and the graph looks the same. Now we must give a precise definition

Two graphs G_1 and G_2 are called **isomorphic** if there exists a one-to-one correspondence between the vertices in G_1 and the vertices in G_2 such that a pair of vertices are adjacent in G_1 if and only if the corresponding pair of vertices are adjacent in G_2

Now we can make "pull vertex 5" story more mathematical: let f be the map between vertices of G_1 and G_2 defined in the following way:

$$f(e) = 5; f(d) = 4, f(b) = 1, f(c) = 2, f(a) = 3.$$

Next it is not hard to check that f is isomorphism, indeed this is one to one map and we can check the property of preserving the adjacency (for example f maps (e, d) to $(5, 4)$).

Two graphs G_1 and G_2 are called **isomorphic** if there exists a one-to-one correspondence between the vertices in G_1 and the vertices in G_2 such that a pair of vertices are adjacent in G_1 if and only if the corresponding pair of vertices are adjacent in G_2

Some almost trivial but useful properties of isomorphic graphs:

Two graphs G_1 and G_2 are called **isomorphic** if there exists a one-to-one correspondence between the vertices in G_1 and the vertices in G_2 such that a pair of vertices are adjacent in G_1 if and only if the corresponding pair of vertices are adjacent in G_2

Some almost trivial but useful properties of isomorphic graphs:

- Number of vertices must be the same.

Two graphs G_1 and G_2 are called **isomorphic** if there exists a one-to-one correspondence between the vertices in G_1 and the vertices in G_2 such that a pair of vertices are adjacent in G_1 if and only if the corresponding pair of vertices are adjacent in G_2

Some almost trivial but useful properties of isomorphic graphs:

- Number of vertices must be the same.
- Number of edges must be the same.

Two graphs G_1 and G_2 are called **isomorphic** if there exists a one-to-one correspondence between the vertices in G_1 and the vertices in G_2 such that a pair of vertices are adjacent in G_1 if and only if the corresponding pair of vertices are adjacent in G_2

Some almost trivial but useful properties of isomorphic graphs:

- Number of vertices must be the same.
- Number of edges must be the same.
- Let $v \in V$ be a vertex. We define $deg(v)$ to be the degree of v (the number of edges incident to the vertex).

Two graphs G_1 and G_2 are called **isomorphic** if there exists a one-to-one correspondence between the vertices in G_1 and the vertices in G_2 such that a pair of vertices are adjacent in G_1 if and only if the corresponding pair of vertices are adjacent in G_2

Some almost trivial but useful properties of isomorphic graphs:

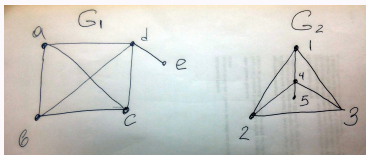
- Number of vertices must be the same.
- Number of edges must be the same.
- Let $v \in V$ be a vertex. We define $deg(v)$ to be the degree of v (the number of edges incident to the vertex). Clearly, the degree should be also preserved under isomorphism (i.e. $deg(v) = deg(f(v))$).

Two graphs G_1 and G_2 are called **isomorphic** if there exists a one-to-one correspondence between the vertices in G_1 and the vertices in G_2 such that a pair of vertices are adjacent in G_1 if and only if the corresponding pair of vertices are adjacent in G_2

Some almost trivial but useful properties of isomorphic graphs:

- Number of vertices must be the same.
- Number of edges must be the same.
- Let $v \in V$ be a vertex. We define $deg(v)$ to be the degree of v (the number of edges incident to the vertex). Clearly, the degree should be also preserved under isomorphism (i.e. $deg(v) = deg(f(v))$).

The degree story helps us to guess the logic of the previous example:

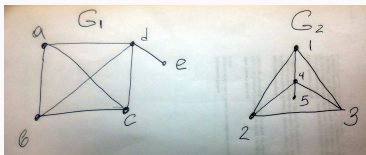


Two graphs G_1 and G_2 are called **isomorphic** if there exists a one-to-one correspondence between the vertices in G_1 and the vertices in G_2 such that a pair of vertices are adjacent in G_1 if and only if the corresponding pair of vertices are adjacent in G_2

Some almost trivial but useful properties of isomorphic graphs:

- Number of vertices must be the same.
- Number of edges must be the same.
- Let $v \in V$ be a vertex. We define $\deg(v)$ to be the degree of v (the number of edges incident to the vertex). Clearly, the degree should be also preserved under isomorphism (i.e. $\deg(v) = \deg(f(v))$).

The degree story helps us to guess the logic of the previous example:



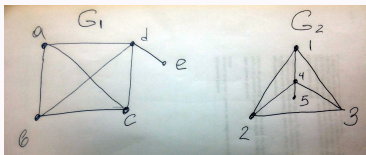
Indeed, $\deg(e) = 1$ and $\deg(5) = 1$ and there are NO other vertices of degree one in those graphs, so the only chance to create isomorphism is to "send" e to 5 , i.e. $f(e) = 5$.

Two graphs G_1 and G_2 are called **isomorphic** if there exists a one-to-one correspondence between the vertices in G_1 and the vertices in G_2 such that a pair of vertices are adjacent in G_1 if and only if the corresponding pair of vertices are adjacent in G_2

Some almost trivial but useful properties of isomorphic graphs:

- Number of vertices must be the same.
- Number of edges must be the same.
- Let $v \in V$ be a vertex. We define $\deg(v)$ to be the degree of v (the number of edges incident to the vertex). Clearly, the degree should be also preserved under isomorphism (i.e. $\deg(v) = \deg(f(v))$).

The degree story helps us to guess the logic of the previous example:



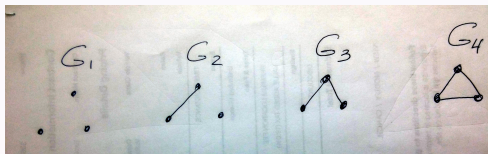
Indeed, $\deg(e) = 1$ and $\deg(5) = 1$ and there are NO other vertices of degree one in those graphs, so the only chance to create isomorphism is to "send" e to 5 , i.e. $f(e) = 5$. The same logic applies to $f(d) = 4$ ($\deg(d) = \deg(4) = 4$ and there are no other vertices of degree 4).

Two graphs G_1 and G_2 are called **isomorphic** if there exists a one-to-one correspondence between the vertices in G_1 and the vertices in G_2 such that a pair of vertices are adjacent in G_1 if and only if the corresponding pair of vertices are adjacent in G_2

We can have a lot of fun with isomorphisms, for example we can list "all graphs" of 3 vertices,

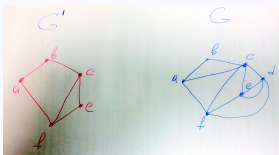
Two graphs G_1 and G_2 are called **isomorphic** if there exists a one-to-one correspondence between the vertices in G_1 and the vertices in G_2 such that a pair of vertices are adjacent in G_1 if and only if the corresponding pair of vertices are adjacent in G_2

We can have a lot of fun with isomorphisms, for example we can list "all graphs" of 3 vertices, more precisely here all of non isomorphic graphs of 3 vertices:



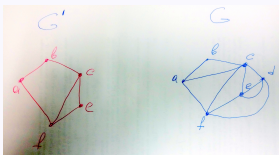
Subgraphs

A subgraph is a graph formed by a subset of vertices and edges of a larger graph.



Subgraphs

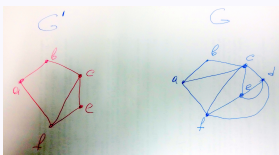
A subgraph is a graph formed by a subset of vertices and edges of a larger graph.



Subgraphs are super useful when we study isomorphisms.

Subgraphs

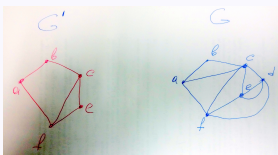
A subgraph is a graph formed by a subset of vertices and edges of a larger graph.



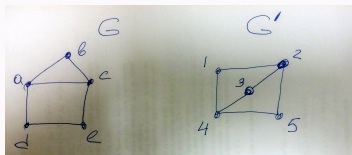
Subgraphs are super useful when we study isomorphisms. Remember, the isomorphisms must preserve "structure" thus must preserve subgraphs.

Subgraphs

A subgraph is a graph formed by a subset of vertices and edges of a larger graph.

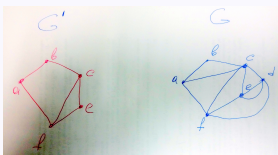


Subgraphs are super useful when we study isomorphisms. Remember, the isomorphisms must preserve "structure" thus must preserve subgraphs. So do you think those two graphs are isomorphic:

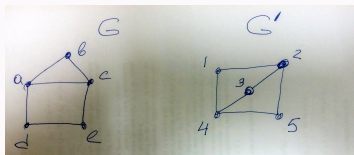


Subgraphs

A subgraph is a graph formed by a subset of vertices and edges of a larger graph.



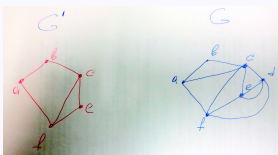
Subgraphs are super useful when we study isomorphisms. Remember, the isomorphisms must preserve "structure" thus must preserve subgraphs. So do you think those two graphs are isomorphic:



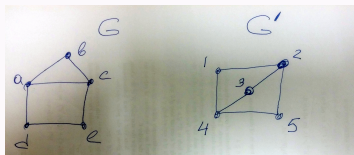
- Vertex count - the same.

Subgraphs

A subgraph is a graph formed by a subset of vertices and edges of a larger graph.

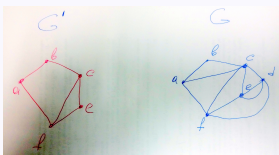


Subgraphs are super useful when we study isomorphisms. Remember, the isomorphisms must preserve "structure" thus must preserve subgraphs. So do you think those two graphs are isomorphic:

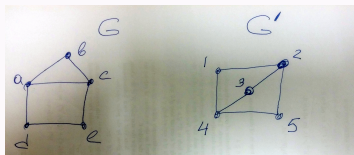


- Vertex count - the same.
- Edge count - the same.

A subgraph is a graph formed by a subset of vertices and edges of a larger graph.



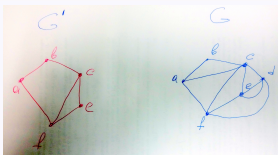
Subgraphs are super useful when we study isomorphisms. Remember, the isomorphisms must preserve "structure" thus must preserve subgraphs. So do you think those two graphs are isomorphic:



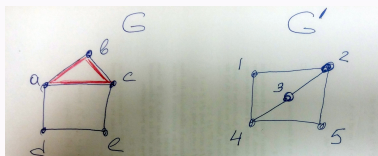
- Vertex count - the same.
- Edge count - the same.
- Degree count is the same. Both graphs 3 vertices of degree 2 and 2 vertices of degree 3.

Subgraphs

A subgraph is a graph formed by a subset of vertices and edges of a larger graph.



Subgraphs are super useful when we study isomorphisms. Remember, the isomorphisms must preserve "structure" thus must preserve subgraphs. So do you think those two graphs are isomorphic:

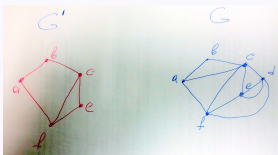


- Vertex count - the same.
- Edge count - the same.
- Degree count is the same. Both graphs 3 vertices of degree 2 and 2 vertices of degree 3.

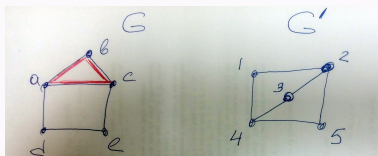
What about now?

Subgraphs

A subgraph is a graph formed by a subset of vertices and edges of a larger graph.



Subgraphs are super useful when we study isomorphisms. Remember, the isomorphisms must preserve "structure" thus must preserve subgraphs. So do you think those two graphs are isomorphic:

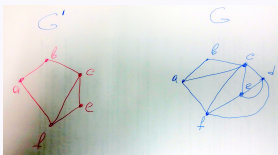


- Vertex count - the same.
- Edge count - the same.
- Degree count is the same. Both graphs 3 vertices of degree 2 and 2 vertices of degree 3.

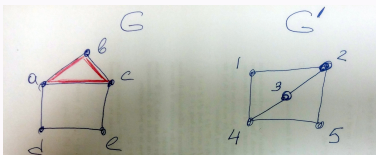
What about now? You may see that G has a subgraph of 3 vertices $\{a, b, c\}$ which is a circuit, there is no circuit of length 3 in G' .

Subgraphs

A subgraph is a graph formed by a subset of vertices and edges of a larger graph.



Subgraphs are super useful when we study isomorphisms. Remember, the isomorphisms must preserve "structure" thus must preserve subgraphs. So do you think those two graphs are isomorphic:

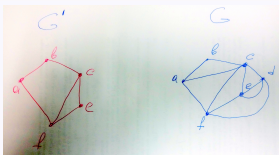


- Vertex count - the same.
- Edge count - the same.
- Degree count is the same. Both graphs 3 vertices of degree 2 and 2 vertices of degree 3.

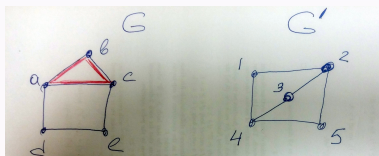
What about now? You may see that G has a subgraph of 3 vertices $\{a, b, c\}$ which is a circuit, there is no circuit of length 3 in G' . So the answer is NO those graphs are not isomorphic.

Subgraphs

A subgraph is a graph formed by a subset of vertices and edges of a larger graph.



Subgraphs are super useful when we study isomorphisms. Remember, the isomorphisms must preserve "structure" thus must preserve subgraphs. So do you think those two graphs are isomorphic:



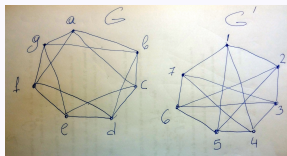
- Vertex count - the same.
- Edge count - the same.
- Degree count is the same. Both graphs 3 vertices of degree 2 and 2 vertices of degree 3.

What about now? You may see that G has a subgraph of 3 vertices $\{a, b, c\}$ which is a circuit, there is no circuit of length 3 in G' . So the answer is NO those graphs are not isomorphic.

NOTICE: this is an example of two graphs, with the same, vertex, edge and degree count which are NOT isomorphic.

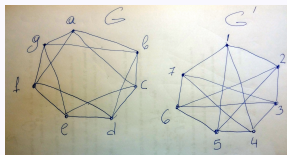
Another example

Are those two graphs isomorphic?



Another example

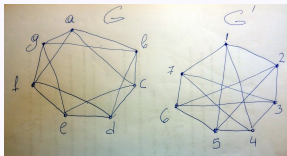
Are those two graphs isomorphic?



- Vertex count - the same.

Another example

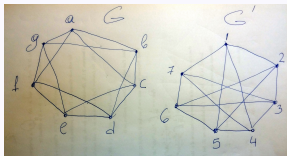
Are those two graphs isomorphic?



- Vertex count - the same.
- Edge count - the same.

Another example

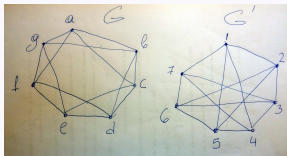
Are those two graphs isomorphic?



- Vertex count - the same.
- Edge count - the same.
- More over, degree count is the same.

Another example

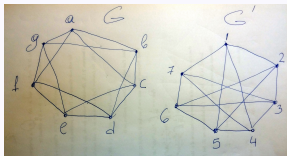
Are those two graphs isomorphic?



- Vertex count - the same.
- Edge count - the same.
- More over, degree count is the same.
- Trying to look for some contradiction in subgraphs no luck.

Another example

Are those two graphs isomorphic?

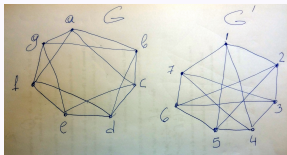


- Vertex count - the same.
- Edge count - the same.
- More over, degree count is the same.
- Trying to look for some contradiction in subgraphs no luck.

So we need just to start working out isomorphism and see how it goes.....

Another example

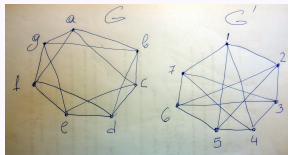
Are those two graphs isomorphic?



So we need just to start working out isomorphism and see how it goes, let's call this isomorphism α :

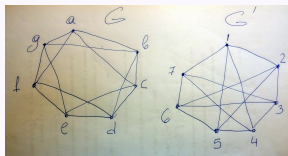
Another example

Are those two graphs isomorphic?



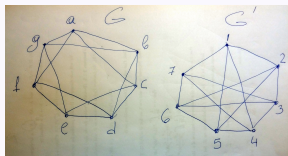
So we need just to start working out isomorphism and see how it goes, let's call this isomorphism α : We notice that in both graphs all vertices are symmetric to each other (they have symmetries of the regular 7-gon).

Are those two graphs isomorphic?



So we need just to start working out isomorphism and see how it goes, let's call this isomorphism α : We notice that in both graphs all vertices are symmetric to each other (they have symmetries of the regular 7-gon). Thus we can send vertex a to any vertex in G' , why not to 1: $\alpha(a) = 1$.

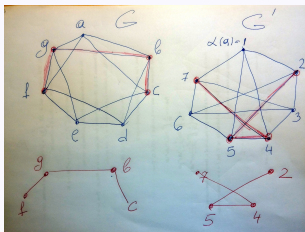
Are those two graphs isomorphic?



So we need just to start working out isomorphism and see how it goes, let's call this isomorphism α : We notice that in both graphs all vertices are symmetric to each other (they have symmetries of the regular 7-gon). Thus we can send vertex a to any vertex in G' , why not to 1: $\alpha(a) = 1$. Now neighbors of a must go to neighbors of 1, but how. Let's see what kind of subgraphs (of neighbors of 1) we get there.

Another example

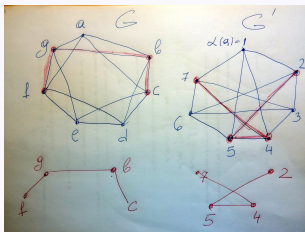
Are those two graphs isomorphic?



So we need just to start working out isomorphism and see how it goes, let's call this isomorphism α : We notice that in both graphs all vertices are symmetric to each other (they show symmetries of the regular 7-gon). Thus we can send vertex a to any vertex in G' why not to 1: $\alpha(a) = 1$. Now neighbors (adjacent vertices) of a must go to neighbors of 1, but how. Let's see what kind of subgraphs (of neighbors of 1) we get there.

Another example

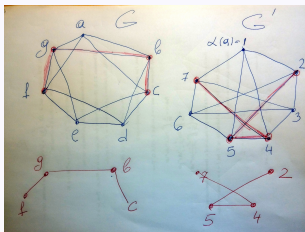
Are those two graphs isomorphic?



So we need just to start working out isomorphism and see how it goes, let's call this isomorphism α : We notice that in both graphs all vertices are symmetric to each other (they show symmetries of the regular 7-gon). Thus we can send vertex a to any vertex in G' why not to 1 : $\alpha(a) = 1$. Now neighbors (adjacent vertices) of a must go to neighbors of 1 , but how. Let's see what kind of subgraphs (of neighbors of 1) we get there. Our isomorphism MUST be also an isomorphism on those subgraphs.

Another example

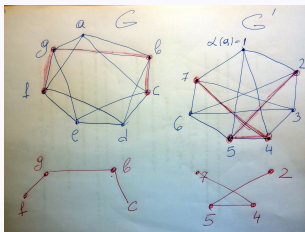
Are those two graphs isomorphic?



So we need just to start working out isomorphism and see how it goes, let's call this isomorphism α : We notice that in both graphs all vertices are symmetric to each other (they show symmetries of the regular 7-gon). Thus we can send vertex a to any vertex in G' why not to 1: $\alpha(a) = 1$. Now neighbors (adjacent vertices) of a must go to neighbors of 1, but how. Let's see what kind of subgraphs (of neighbors of 1) we get there. Our isomorphism MUST be also an isomorphism on those subgraphs. But when we look at them we see that there is no much choice: f which is of degree 1 must go to 7 or to 2 and (again by symmetry) there is no much difference to which of them. Set $\alpha(f) = 7$, then automatically: $\alpha(g) = 4, \alpha(b) = 4, \alpha(c) = 2$.

Another example

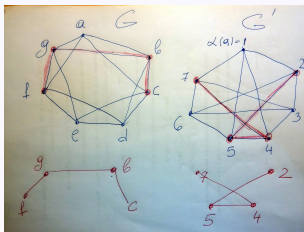
Are those two graphs isomorphic?



So we need just to start working out isomorphism and see how it goes, let's call this isomorphism α : We notice that in both graphs all vertices are symmetric to each other (they show symmetries of the regular 7-gon). Thus we can send vertex a to any vertex in G' why not to 1 : $\alpha(a) = 1$. Now neighbors (adjacent vertices) of a must go to neighbors of 1 , but how. Let's see what kind of subgraphs (of neighbors of 1) we get there. Our isomorphism MUST be also an isomorphism on those subgraphs. But when we look at them we see that there is no much choice: f which is of degree 1 must go to 7 or to 2 and (again by symmetry) there is no much difference to which of them. Set $\alpha(f) = 7$, then automatically: $\alpha(g) = 4, \alpha(b) = 4, \alpha(c) = 2$. We left to decide about vertices e and d and $3, 6$.

Another example

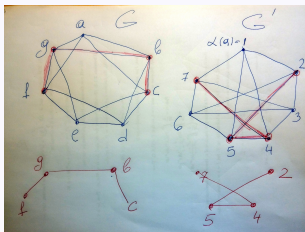
Are those two graphs isomorphic?



So we need just to start working out isomorphism and see how it goes, let's call this isomorphism α : We notice that in both graphs all vertices are symmetric to each other (they show symmetries of the regular 7-gon). Thus we can send vertex a to any vertex in G' why not to 1: $\alpha(a) = 1$. Now neighbors (adjacent vertices) of a must go to neighbors of 1, but how. Let's see what kind of subgraphs (of neighbors of 1) we get there. Our isomorphism MUST be also an isomorphism on those subgraphs. But when we look at them we see that there is no much choice: f which is of degree 1 must go to 7 or to 2 and (again by symmetry) there is no much difference to which of them. Set $\alpha(f) = 7$, then automatically: $\alpha(g) = 4, \alpha(b) = 4, \alpha(c) = 2$. We left to decide about vertices e and d and $3, 6$. Notice that g is adjacent to e , and $\alpha(g) = 4$,

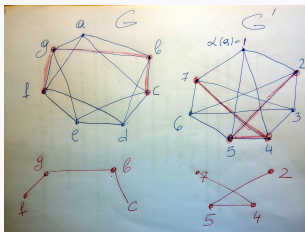
Another example

Are those two graphs isomorphic?



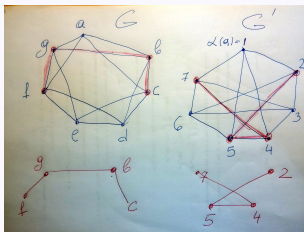
So we need just to start working out isomorphism and see how it goes, let's call this isomorphism α : We notice that in both graphs all vertices are symmetric to each other (they show symmetries of the regular 7-gon). Thus we can send vertex a to any vertex in G' why not to 1 : $\alpha(a) = 1$. Now neighbors (adjacent vertices) of a must go to neighbors of 1 , but how. Let's see what kind of subgraphs (of neighbors of 1) we get there. Our isomorphism MUST be also an isomorphism on those subgraphs. But when we look at them we see that there is no much choice: f which is of degree 1 must go to 7 or to 2 and (again by symmetry) there is no much difference to which of them. Set $\alpha(f) = 7$, then automatically: $\alpha(g) = 4$, $\alpha(b) = 4$, $\alpha(c) = 2$. We left to decide about vertices e and d and $3, 6$. Notice that g is adjacent to e , and $\alpha(g) = 4$, thus $\alpha(e) \neq 6$ because 6 is not adjacent to 4

Are those two graphs isomorphic?



So we need just to start working out isomorphism and see how it goes, let's call this isomorphism α : We notice that in both graphs all vertices are symmetric to each other (they show symmetries of the regular 7-gon). Thus we can send vertex a to any vertex in G' why not to 1: $\alpha(a) = 1$. Now neighbors (adjacent vertices) of a must go to neighbors of 1, but how. Let's see what kind of subgraphs (of neighbors of 1) we get there. Our isomorphism MUST be also an isomorphism on those subgraphs. But when we look at them we see that there is no much choice: f which is of degree 1 must go to 7 or to 2 and (again by symmetry) there is no much difference to which of them. Set $\alpha(f) = 7$, then automatically: $\alpha(g) = 4, \alpha(b) = 4, \alpha(c) = 2$. We left to decide about vertices e and d and $3, 6$. Notice that g is adjacent to e , and $\alpha(g) = 4$, thus $\alpha(e) \neq 6$ because 6 is not adjacent to 4 and the only choice $\alpha(e) = 3$ and $\alpha(d) = 6$.

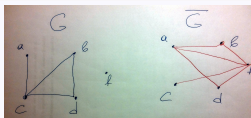
Are those two graphs isomorphic?



So we need just to start working out isomorphism and see how it goes, let's call this isomorphism α : We notice that in both graphs all vertices are symmetric to each other (they show symmetries of the regular 7-gon). Thus we can send vertex a to any vertex in G' why not to 1: $\alpha(a) = 1$. Now neighbors (adjacent vertices) of a must go to neighbors of 1, but how. Let's see what kind of subgraphs (of neighbors of 1) we get there. Our isomorphism MUST be also an isomorphism on those subgraphs. But when we look at them we see that there is no much choice: f which is of degree 1 must go to 7 or to 2 and (again by symmetry) there is no much difference to which of them. Set $\alpha(f) = 7$, then automatically: $\alpha(g) = 4, \alpha(b) = 4, \alpha(c) = 2$. We left to decide about vertices e and d and $3, 6$. Notice that g is adjacent to e , and $\alpha(g) = 4$, thus $\alpha(e) \neq 6$ because 6 is not adjacent to 4 and the only choice $\alpha(e) = 3$ and $\alpha(d) = 6$. The final steps is to check that all adjacency relations are preserved.

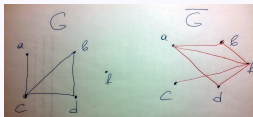
Yet, another example + a cool idea

Consider a graph $G = (V, E)$, its **complement** is a graph $\bar{G} = (V, E')$ with the same set of vertices but now with edges between exactly those pairs of vertices that were not adjacent in G .



Yet, another example + a cool idea

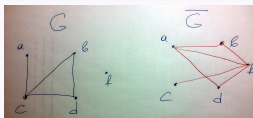
Consider a graph $G = (V, E)$, its **complement** is a graph $\bar{G} = (V, E')$ with the same set of vertices but now with edges between exactly those pairs of vertices that were not adjacent in G .



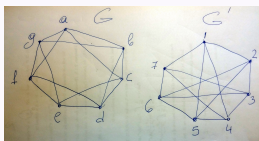
Notice that, if G_1 and G_2 are isomorphic if and only if \bar{G}_1 and \bar{G}_2 are isomorphic (you actually may use the same isomorphism).

Yet, another example + a cool idea

Consider a graph $G = (V, E)$, its **complement** is a graph $\bar{G} = (V, E')$ with the same set of vertices but now with edges between exactly those pairs of vertices that were not adjacent in G .

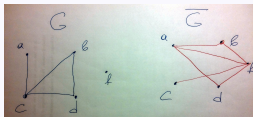


Notice that, if G_1 and G_2 are isomorphic if and only if \bar{G}_1 and \bar{G}_2 are isomorphic (you actually may use the same isomorphism). Consider previous example

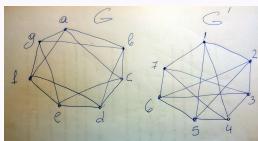


Yet, another example + a cool idea

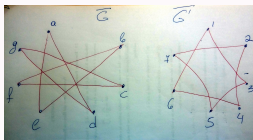
Consider a graph $G = (V, E)$, its **complement** is a graph $\bar{G} = (V, E')$ with the same set of vertices but now with edges between exactly those pairs of vertices that were not adjacent in G .



Notice that, if G_1 and G_2 are isomorphic if and only if \bar{G}_1 and \bar{G}_2 are isomorphic (you actually may use the same isomorphism). Consider previous example

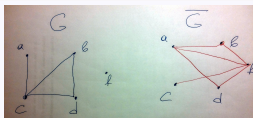


Now lets draw a complement graphs

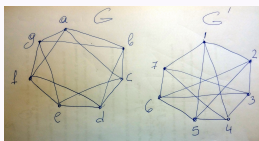


Yet, another example + a cool idea

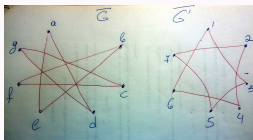
Consider a graph $G = (V, E)$, its **complement** is a graph $\bar{G} = (V, E')$ with the same set of vertices but now with edges between exactly those pairs of vertices that were not adjacent in G .



Notice that, if G_1 and G_2 are isomorphic if and only if \bar{G}_1 and \bar{G}_2 are isomorphic (you actually may use the same isomorphism). Consider previous example



Now lets draw a complement graphs



Both of them are circuits of length 7, thus isomorphic, and solution became almost trivial with this cool trick!!!