

Lecture 7

MATH-42021/52021 Graph Theory and Combinatorics.

Artem Zvavitch

Department of Mathematical Sciences, Kent State University

June, 2016.

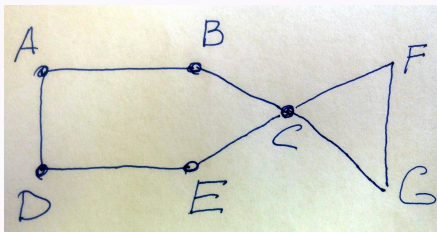
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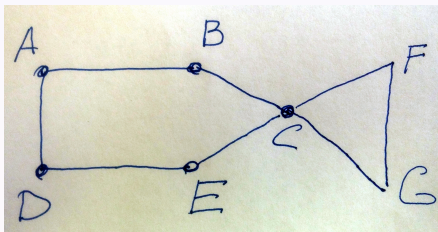
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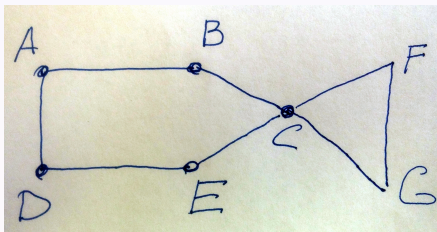
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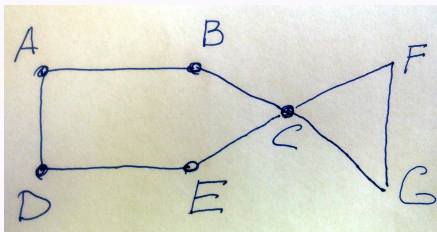
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How to determine if it is possible or not? Is it possible to create an algorithm? How fast such an algorithm could be?

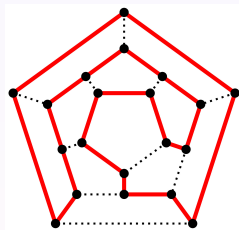
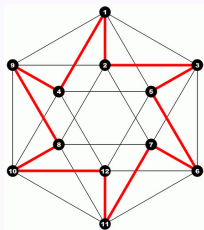
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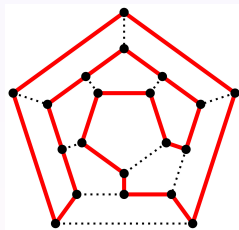
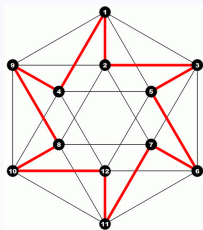
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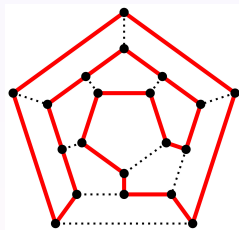
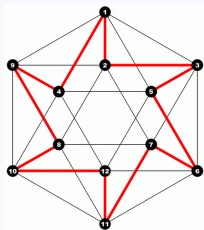
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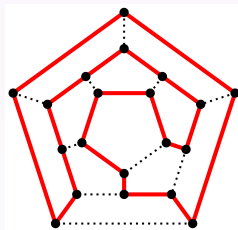
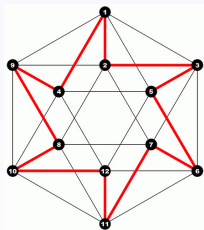
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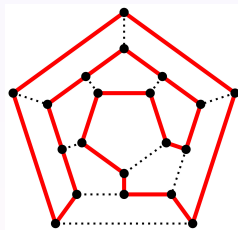
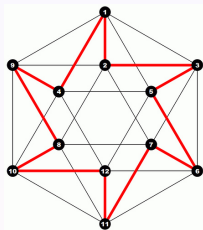
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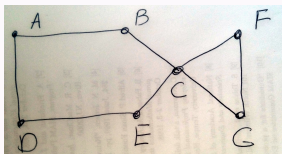
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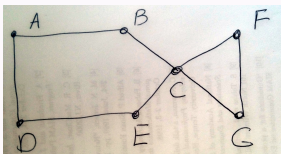
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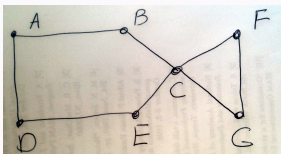
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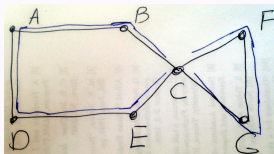
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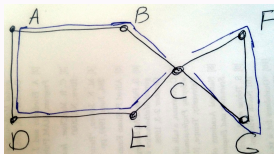
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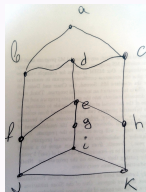
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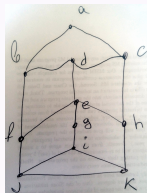
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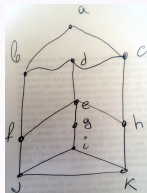
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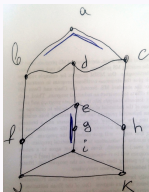
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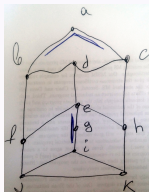
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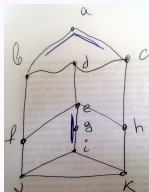
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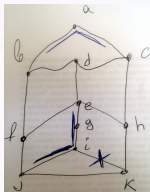
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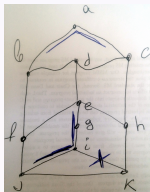
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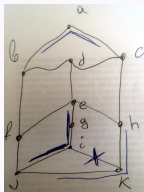
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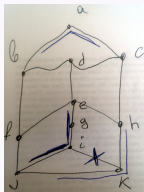
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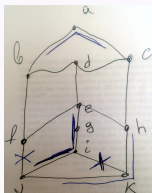
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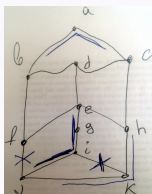
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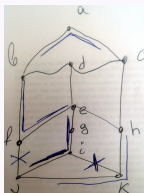
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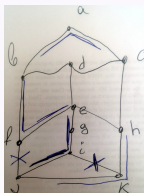
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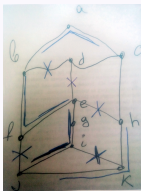
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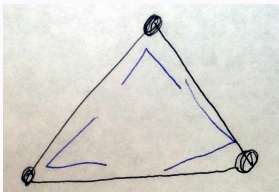
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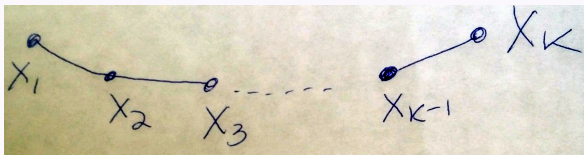
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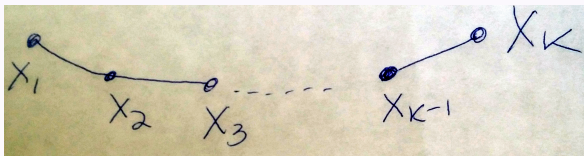
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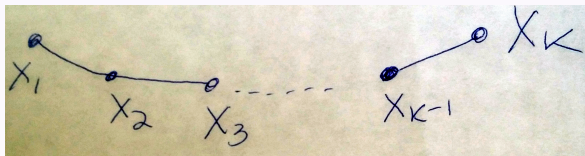


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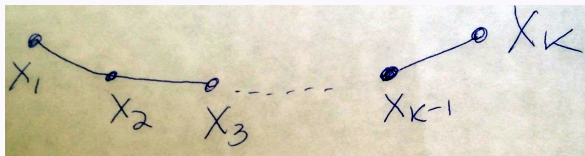


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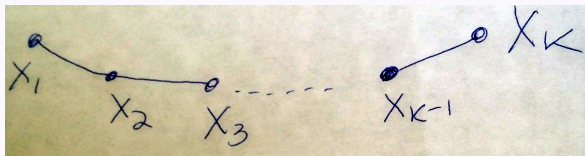


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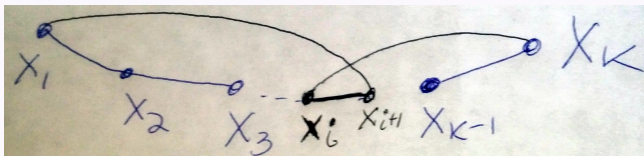


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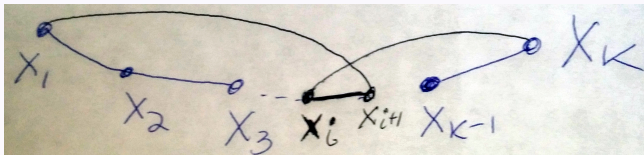


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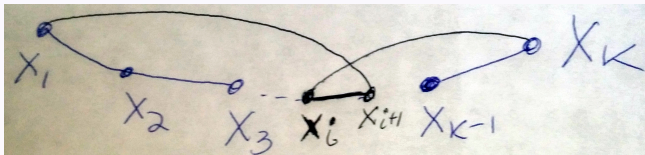


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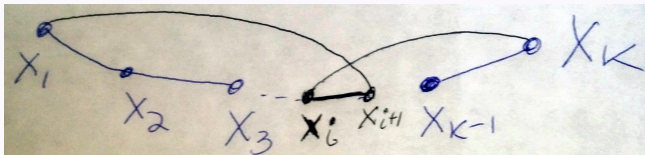


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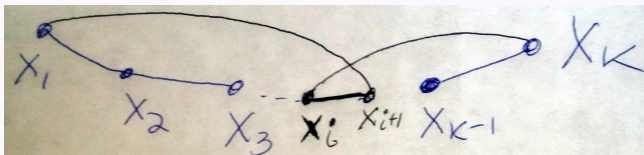
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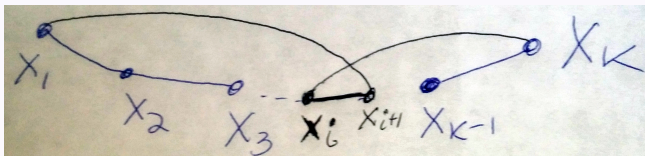
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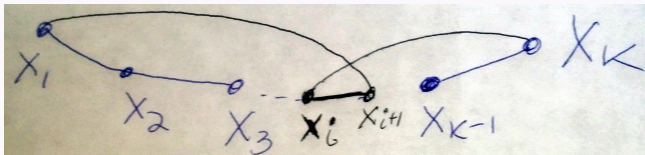
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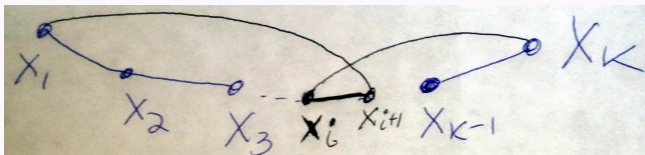
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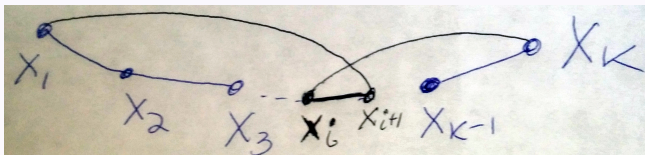
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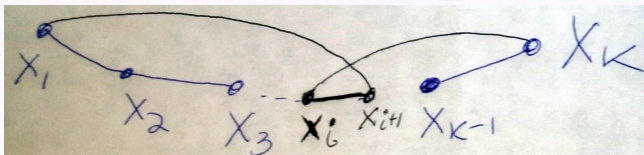
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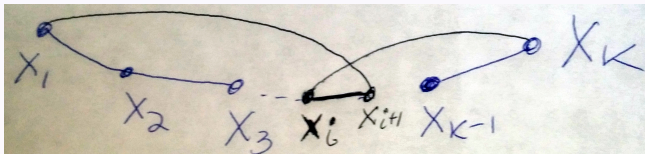
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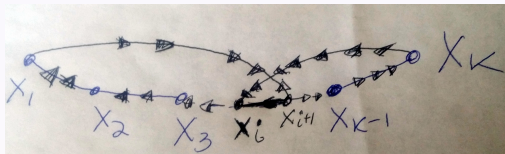
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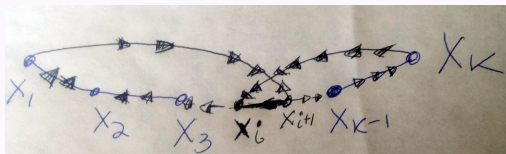
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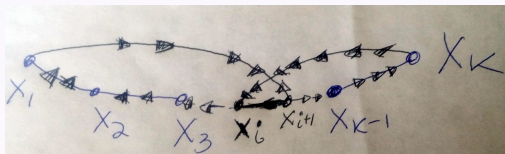
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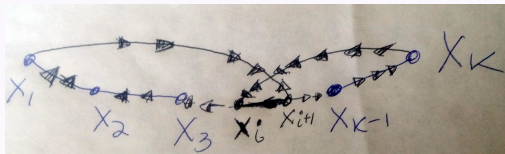
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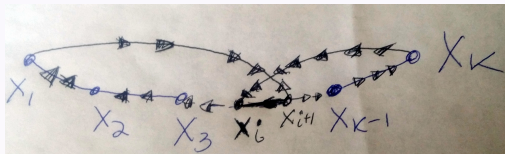
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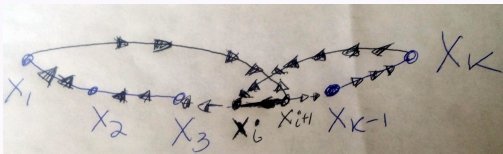
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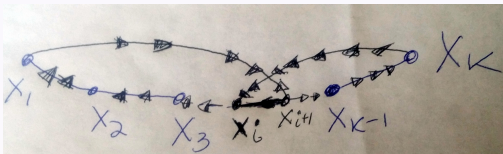
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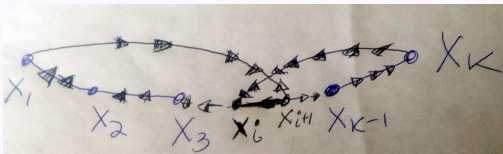
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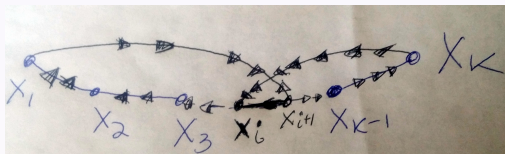
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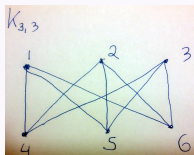
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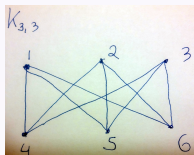
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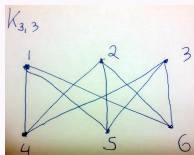


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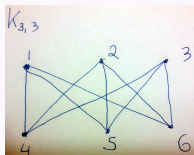


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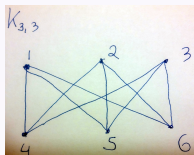
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