Lecture 7
MATH-42021/52021 Graph Theory and Combinatorics.

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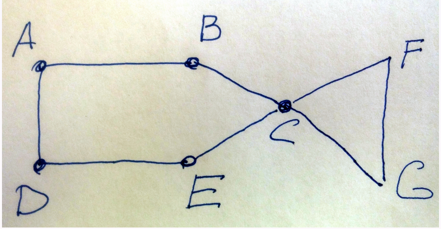

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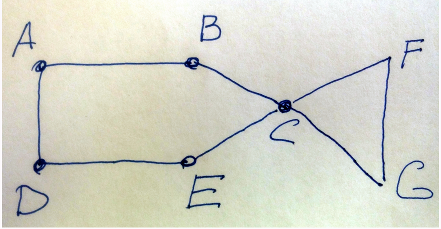

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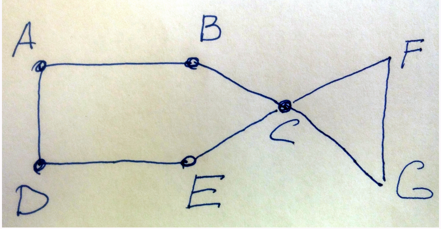

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Salesman Problem

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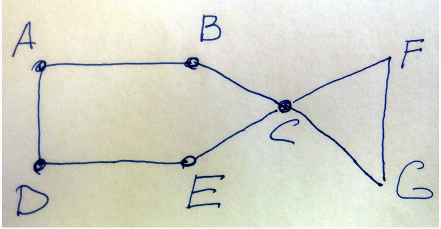

How to determine if it is possible or not? Is it possible to create an algorithm? How fast such an algorithm could be?

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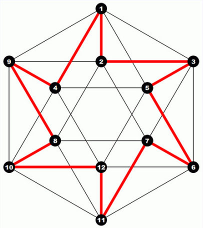

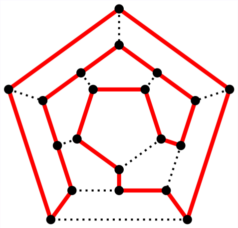

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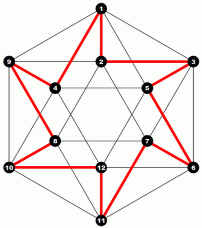

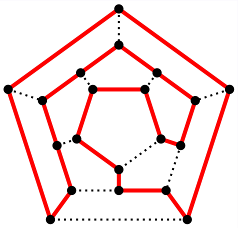

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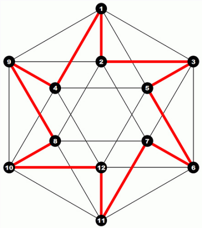

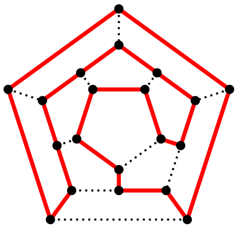

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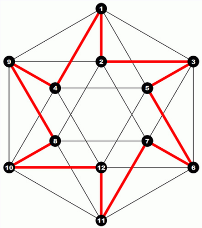

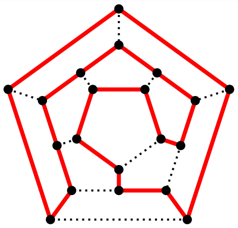

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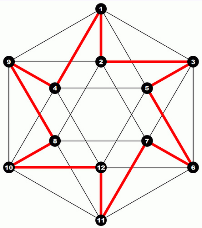

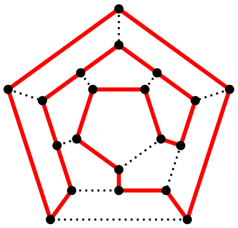

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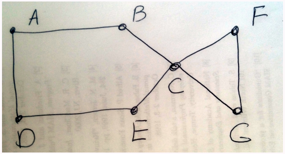

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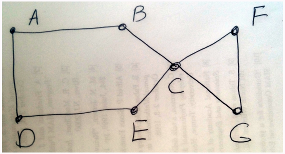

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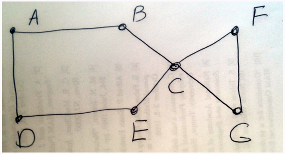

has no Hamilton circuit. We start by applying Rule 1. We see that the vertices A, B, F, G, E, D are all of degree two. Thus is Hamilton circuit exist it must use all edges adjacent to them. We mark those edges (better to say "vertices/corners") with additional lines/turns.

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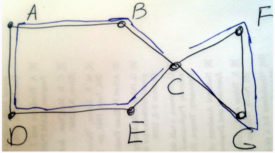

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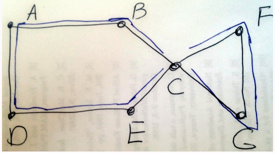

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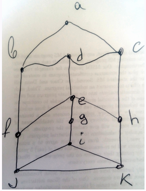

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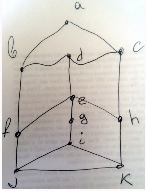

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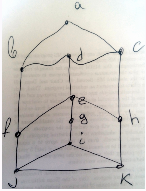

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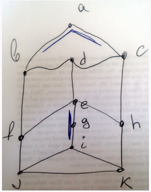

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Hamilton Circuits – some rules for possible existence / one more example

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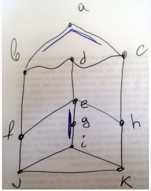

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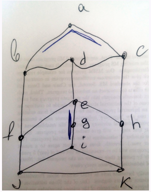

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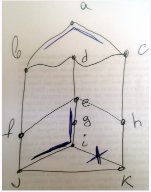

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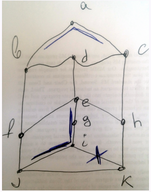

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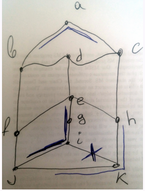

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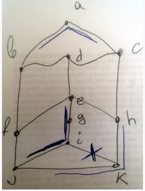

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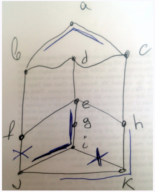

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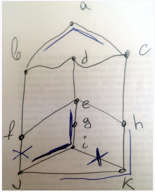

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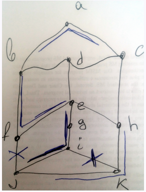

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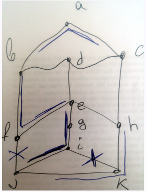

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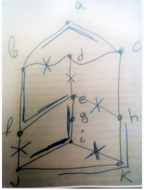

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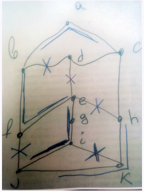

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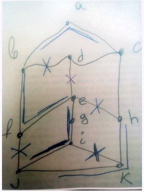

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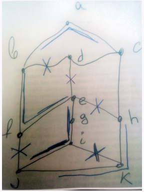

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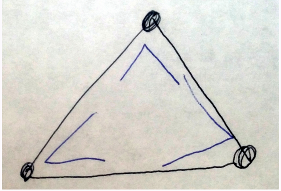

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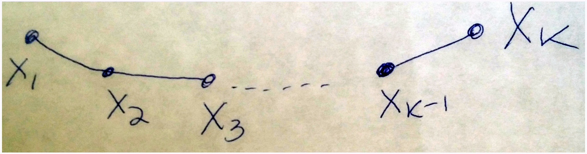

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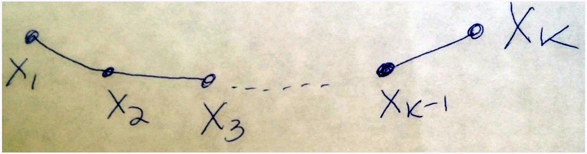

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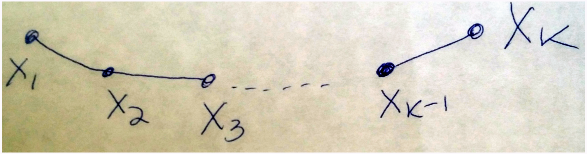

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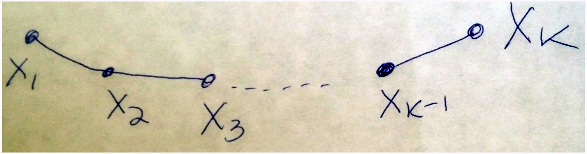

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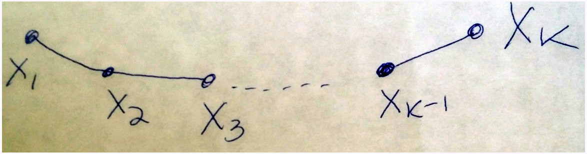

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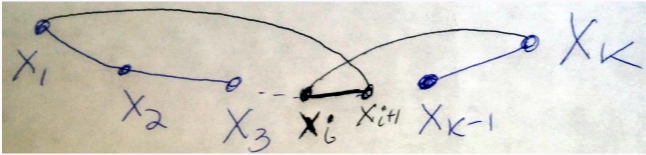

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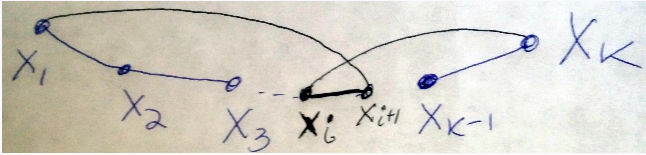

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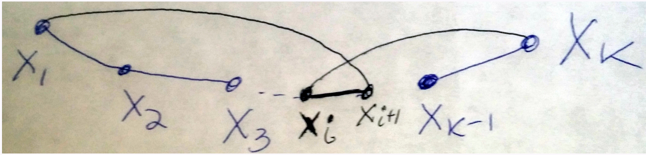

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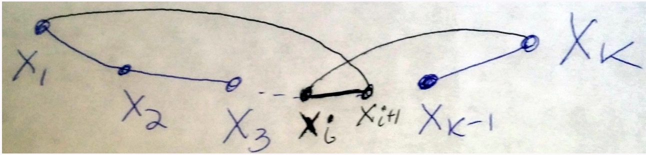

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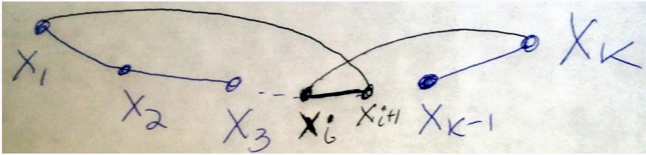

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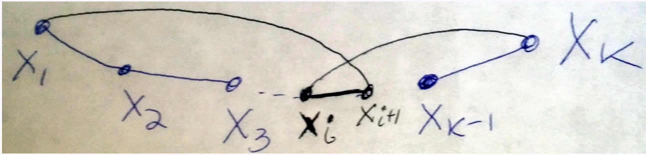

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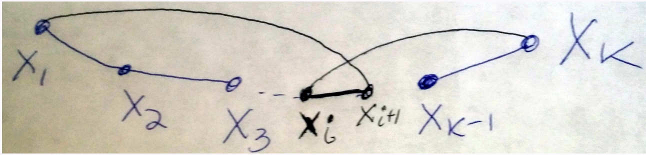

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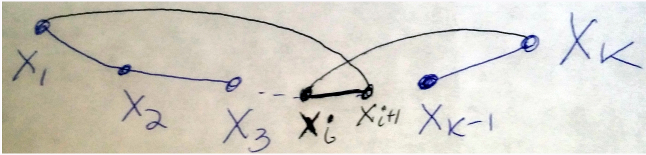

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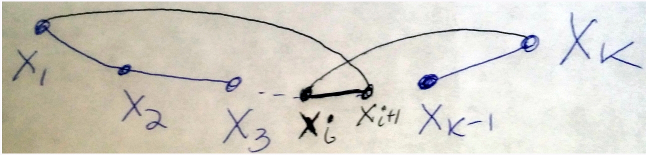

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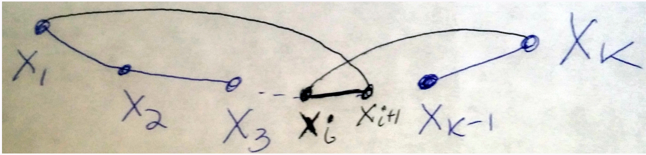

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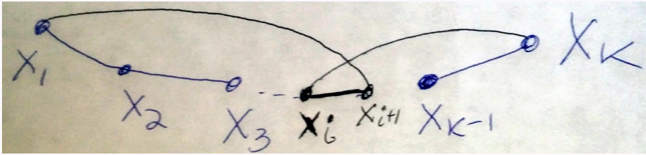

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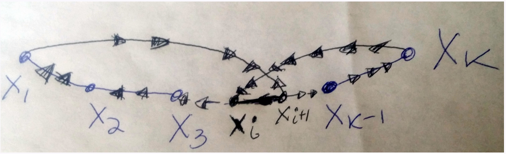

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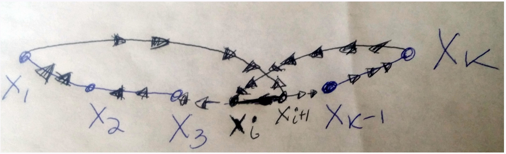

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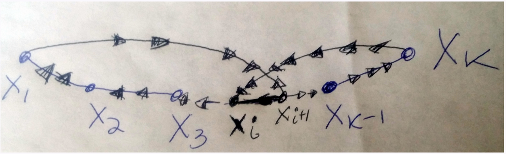

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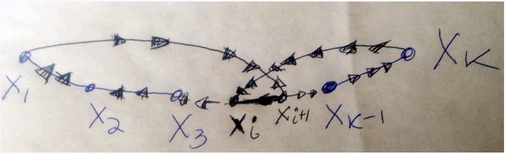

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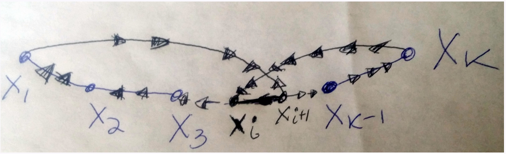

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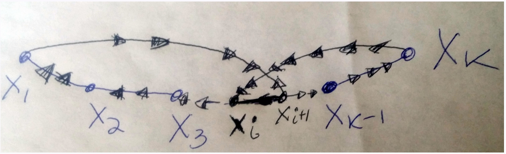

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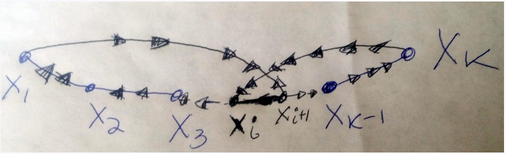

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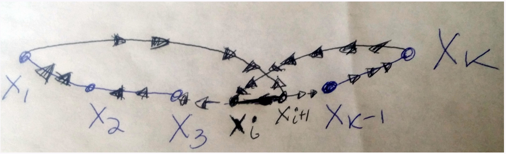

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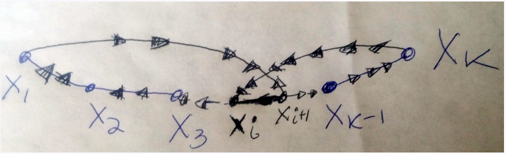

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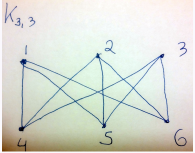
Theorem is quite useful, for example our old friend $K_{3,3}$



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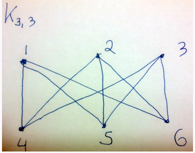

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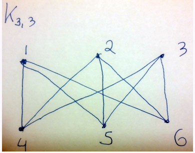

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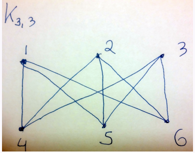

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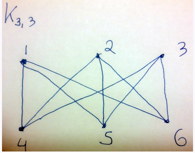

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