

# DARBOUX TRANSFORMATION AND ORTHOGONAL POLYNOMIALS: MATRIX INTERPRETATION AND NUMERICAL ASPECTS

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Darboux transformations, also known as spectral transformations of orthogonal polynomials, were first studied by Christoffel and Geronimus in 1858 and 1940, respectively. If  $L$  denotes a moment functional with integral representation in terms of a measure  $\mu$ , then Christoffel transformation consider a polynomial perturbation of the corresponding measure while Geronimus consider a rational perturbation of the measure. If  $\{P_n\}$  denotes a sequence of orthogonal polynomials with respect to  $L$ , and a Darboux transformation is applied to  $L$ , new sequences of orthogonal polynomials arise. In the literature, the algebraic relation between both sequences of polynomials can be found.

The papers by Geronimus were not properly estimated by mathematicians after their publication. However, in the 90's, they attracted attention of various specialists in different branches of mathematics and mathematical physics. Darboux transformations have wide applications nowadays in quantum mechanics, theory of discrete integrable systems, computation of Gaussian quadrature rules, bispectral problem in theory of orthogonal polynomials, and so on.

The sequence of orthogonal polynomials with respect to  $L$  satisfies a three-term recurrence relation whose coefficients are the entries of the so-called monic Jacobi matrix. We give a matrix interpretation of the Darboux transformations, that is, we show how to find the monic Jacobi matrix associated with the Darboux transforms of  $L$  in terms of the monic Jacobi matrix associated with  $L$ . The main tool used is LU and UL factorizations. Since the eigenvalues of the  $n$ th leading principal submatrix of the Jacobi matrix are the zeros of the orthogonal polynomial of degree  $n$ , the study of the numerical properties of the algorithms for computing the transformed Jacobi matrix arises as a natural problem. We present the first formal study of the stability and conditioning of the algorithm that computes one of the Darboux transformations. The main result is that this algorithm is forward stable.

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