

GENERALIZATIONS OF VERTEX ALGEBRAS

M. ROITMAN

A vertex algebra is, roughly speaking, a linear space A that has infinitely many bilinear operations $(a, b) \rightarrow a(n)b$, indexed by integer number n , satisfying certain identities. It is natural to consider the generating function for these products:

$$Y(a, z) = \dots + a(-2)z + a(-1) + a(0)z^{-1} + a(1)z^{-2} + \dots$$

where $a(n) : A \rightarrow A$ is the operator of n -th left multiplication by a . One of the main properties of vertex algebras is that for any $a_1, \dots, a_k, b \in A$ and a functional $f : A \rightarrow C$, the series

$$f(Y(a_1, z_1)Y(a_2, z_2)\dots Y(a_l, z_k)b), \quad a_i, b \in A$$

converge in some region of \mathbb{C}^k to a rational function in z_1, \dots, z_k . We consider a more general class of algebras, such that the above series converge to an analytic function in k variables, which is not necessarily rational. We outline how these structures appear in representation theory and geometry, and give a method of constructing such algebras starting from certain spaces of complex analytic functions.

DEPARTMENT OF MATHEMATICS,, UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
E-mail address: mroitman@yahoo.com