Problem 1. Prove that 
\[ 1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}. \]

Problem 2. Prove that 
\[ \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} \geq \sqrt{n}, \]
for all \( n \in \mathbb{N} \).

Problem 3. (Extra) Prove that for \( n \geq 5 \)
\[ \frac{n^n}{3^n} \leq n! \leq \frac{n^n}{2^n}. \]

Problem 4. Consider a set \( A \). We say \#A = 0 if \( A \) is the empty set and we say that \#A = n if there is a bijection \( f : A \to \{1, \ldots, n\} \). We say \( A \) is finite if \( A \) is empty or \#A = n for some natural number \( n \). Please, show that \#A is well defined, i.e. for finite set \( A \) there is only one number \( n \) such that \#A = n. (hint: You may use a book or any other source)

Problem 5. Let \( A \) and \( B \) be a countable sets show that \( A \cap B \) is also a countable set.

Problem 6. Consider a set \( S \) whose elements are nonoverlapping intervals of length 1 (i.e. for any \([a_1, b_1] \in S \) and \([a_2, b_2] \in S \) \([a_1, b_1] \cap [a_2, b_2] = \emptyset \) and \( b_1 - a_1 = b_2 - a_2 = 1 \)). PLEASE SHOW THAT \( S \) is a countable set.