Problem 1. Consider an interval I = [a, b] ⊂ R and a differentiable function f on I. Assume that f’ is bounded on I, prove that f is a Lipschitz function.

Problem 2. Consider a Lipschitz function f on I = [a, b] ⊂ R is it true that f is differentiable on I.

Problem 3. Find an example of a function which is differentiable on [-1, 1], but not twice differentiable on [-1, 1].

Problem 4. Prove that the derivative of the function
\[ f(x) = e^x(x - 1)(x - 2)(x - 3)(x - 4)(x - 5) \]
has at least four roots.

Problem 5. Find
- \( \lim_{x \to 0} \frac{\sin x^2}{x} \).
- \( \lim_{x \to \infty} \frac{\log x}{e^x} \).
- \( \lim_{x \to 0^+} (\sin x)^x \).

Problem 6. Show that if \( x > 0 \), then
\[ 1 + \frac{1}{2}x - \frac{1}{8}x^2 \leq \sqrt{1 + x} \leq 1 + \frac{1}{2}x. \]

Problem 7. Show that the sum of two convex functions is again a convex function. Is the same statement true for product? Construct a convex function defined on R which is increasing for all \( x \in R \).