Introduction to Analysis 2
Home Work 3, due Monday, February 22.
Instructor: Prof. Artem Zvavitch

Problem 1. Please construct a function which is infinitely differentiable on \( \mathbb{R} \) (i.e. \( f^{(n)}(x) \) exists for all \( n \in \mathbb{N} \) and \( x \in \mathbb{R} \)), such that \( f^{(n)}(0) = 0 \) for all \( n \), but \( f(x) \neq 0 \). Explain why this example do not contradict Taylor’s theorem.

Problem 2. Compute \( e \) correct to 6 decimal place.

Problem 3. Use the definition of integral to show that function \( f(x) = 1 \) for \( x \in [0,1] \) and \( f(x) = 3x \) for \( x \in (1,2] \) is integrable on \([0,2]\).

Problem 4. Show that the function \( f(x) = 0 \) for \( x = 0 \) and \( f(x) = 1/x \) for \( x \in (0,1] \) is not integrable function on \([0,1]\).

Problem 5. Let \( f \) be a Riemann integrable function on \([a,b]\). Consider function \( g(x) \) on \([a,b]\) such that \( g(x) = f(x) \) for all except finite number of points \( c_1, c_2, ..., c_n \in [a,b] \). Prove that \( g(x) \) is Riemann integrable and

\[
\int_{a}^{b} g dx = \int_{a}^{b} f dx.
\]

Problem 6. Let \( f \) be a Riemann integrable function on \([a,b]\). Consider function \( g(x) \) on \([a,b]\) such that \( g(x) = f(x) \) for all except COUNTABLE set of points \( c_1, c_2, c_3, ... \in [a,b] \). Show that \( g(x) \) is not necessary integrable on \([a,b]\).