Problem 1. Prove that the following sequences are convergent and find the limits (you may use ANY results from class/book)

(1) \( b_1 = 8, b_{n+1} = \frac{1}{2}b_n + 2 \).
(2) \( (1 + \frac{1}{n})^{n+1} \).
(3) \( (1 - \frac{1}{n})^n \).
(4) \( (1 + \frac{1}{n})^{4n^3} \).
(5) \( (1 + \frac{1}{2n})^7 \).

Problem 2. Prove that the following sequence is convergent

\[ a_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!} \].

Problem 3. Consider a sequence \((a_n : n \in N)\), such that

\[ |a_{n+1} - a_n| \leq \frac{1}{3^n}. \]

Show that \((a_n : n \in N)\) is convergent.

Problem 4. Assume \(a_1 < a_2\) are arbitrary real numbers and \(a_n = \frac{1}{2}(a_{n-2} + a_{n-1})\), for \(n > 2\), show that \(a_n\) is a Cauchy sequence. Find the limit.

Problem 5. Is it true that every bounded sequence is Cauchy sequence? Is it true that every bounded sequence has a Cauchy subsequence?