Problem 1. Use \( \varepsilon - \delta \) definition to show the continuity of function \( f(x) = \frac{1}{x+1} \) at point \( x = 2 \).

Problem 2. Suppose that \( f \) is a continuous function on \([0, 1]\) such that \( f(r) = 0 \) for every rational number \( r \). Prove that \( f(x) = 0 \) for all \( x \in [0, 1] \).

Problem 3. Suppose that \( f \) and \( g \) are continuous function on \([0, 1]\) such that \( f(r) = g(r) \) for every rational number \( r \). Prove that \( f(x) = g(x) \) for all \( x \in [0, 1] \).

Problem 4. Show that the function
\[
f(x) = \frac{\sqrt{x+1}}{\sqrt{1+\sqrt{x}}}
\]
is a continuous function for \( x \in [0, \infty) \).

Problem 5. Show an example of bounded set \( A \) and a function \( f \) continuous on \( A \), such that \( f \) is unbounded function on \( A \).

Problem 6. Prove that if
\[
|f(x) - f(y)| \leq |x - y|^2
\]
for all \( x, y \) in \([0, 1]\) then \( f(x) \) is a continues function on \([0, 1]\).