Problem 1. Solve heat equation \( \frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} \) \( , \ x \in [0, L] \), \( t > 0 \). subject to \( u(0, t) = 0 \), \( u(L, t) = 0 \) and
\[
\left\{
\begin{array}{ll}
0 & \text{if } 0 < x < L/2 \\
2 & \text{if } L/2 < x < L
\end{array}
\right.
\]
\( u(x, 0) = 2 \cos \frac{3\pi x}{L} \). 

Problem 2. Solve heat equation \( \frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} \) subject to \( \frac{\partial u}{\partial x} u(0, t) = 0 \), \( \frac{\partial u}{\partial x} u(L, t) = 0 \) and initial condition
\[
\left\{
\begin{array}{ll}
1 & \text{if } 0 < x < L/2 \\
2 & \text{if } L/2 < x < L
\end{array}
\right.
\]

Problem 3. Solve the eigenvalue problem
\[
\phi''(x) = -\lambda \phi(x)
\]
subject to \( \phi(0) = \phi(2\pi) \) and \( \phi'(0) = \phi'(2\pi) \).

Problem 4. Solve Laplace’s equation \( \triangle u(x, y) = 0 \) inside rectangle \( [0, L] \times [0, H] \), subject to
\[
\frac{\partial u}{\partial x}(0, y) = 0, u(L, y) = g(y), u(x, 0) = 0, u(x, H) = 0.
\]

Problem 5. Solve Laplace’s equation outside a circular disk of radius \( a \) (i.e. \( r \geq a \)) subject to the boundary condition \( u(a, \theta) = \ln 2 + 4 \cos 3\theta \).