# INFORMAL ANALYSIS SEMINAR:
## Differentiation in finite & infinite dimensions
## Kent State University April 7 - 9, 2017

*We are very grateful to Elsevier and also to various offices in Kent State University (Mathematics, Arts & Sciences, Research) for generous support.*

Welcome to the Informal Analysis Seminar. These pages contain information that might prove helpful and useful during the meeting. Here's a rough schedule of talks. Note that Abstracts are attached, in more or less alphabetical order. All talks are in Room 228.

Friday afternoon: 3 PM Daniel Azagra ("Whitney extension theorems for convex functions")
+/- 4 – 4:20 PM Refreshments
+/- 4:20 PM Vladimir Peller ("Schatten - von Neumann properties of multiple operator integrals and Lipschitz type estimates for functions of triples of self-adjoint operators")
+/- 5:30 PM Estabilitz Durand-Cartagena ("AMLE's and infinity-harmonic functions in metric measure spaces")

*All participants are warmly invited to a reception at Eleanor and Richard's house, from around 7 PM. It's around a 20 minute walk to our house or a 5 - 10 minute drive (provided one goes in the right direction). Please refer to the attached map for location.*

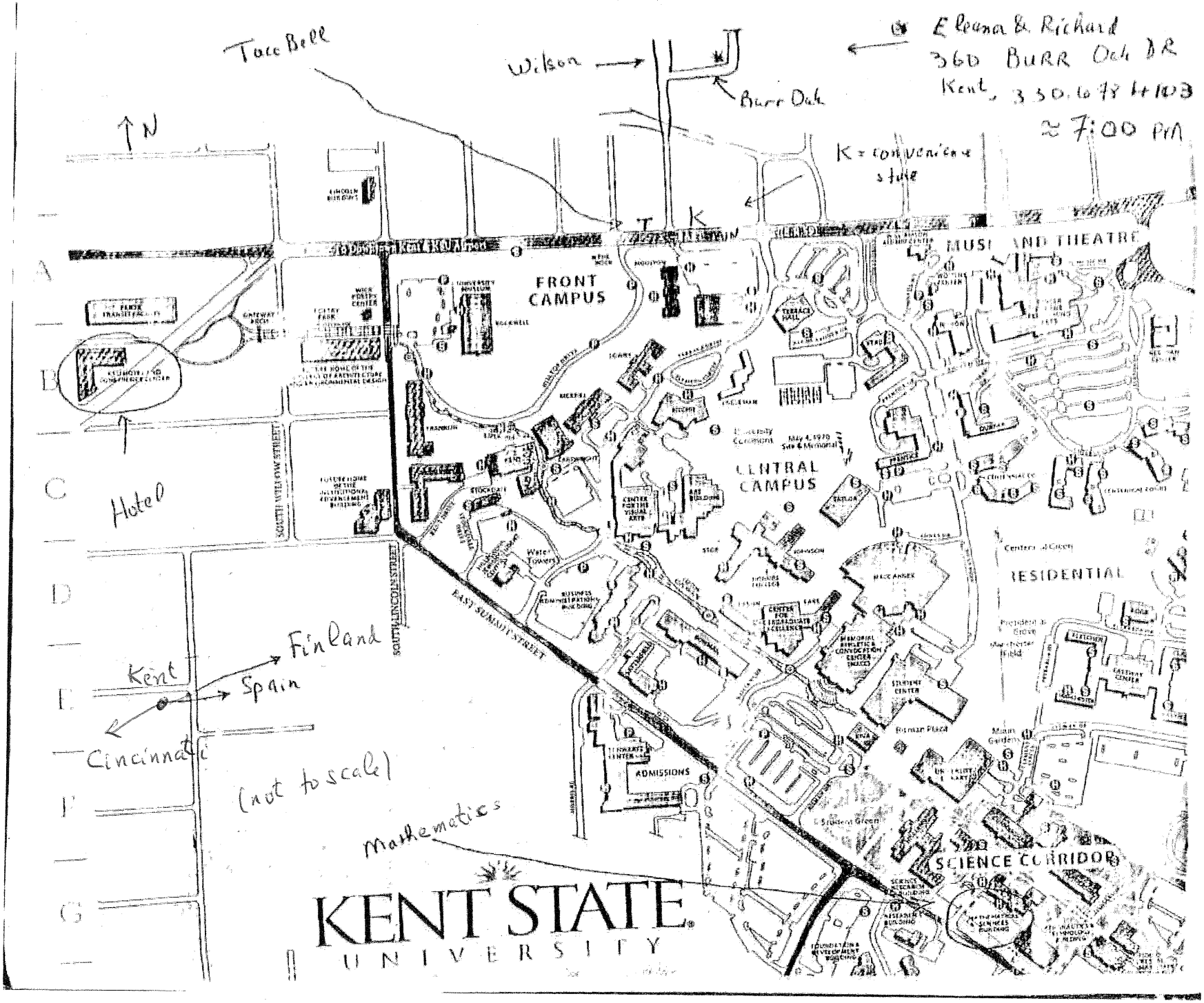
Saturday morning: 9:30 AM Jesus Jaramillo ("Inversion of non-smooth mappings in Banach spaces")
+/- 10:30 AM Coffee
+/- 11:00 AM Nageswari Shanmugalingam ("The p-Harmonic versus 1-harmonic Dirichlet problem")
+/- 12:00 —2 PM Lunch + posters. (Lunch will be on the top floor of the building. *Note that posters will be on display throughout the meeting and, in particular, presenters of posters will be on hand during lunch to discuss their work.*

Saturday afternoon: 2:00 PM Patrick Rabier ("Boundedness of functions with $p$-integrable gradients")
+/- 3:00 PM Piotr Hajlasz ("A measure and orientation preserving homeomorphism with approximate Jacobian equal -1 almost everywhere")
+/- 4:00 PM Refreshments
+/- 4:30 PM Pekka Koskela ("Planar Sobolev extension domains")

*Dinner will be at 7 PM in* Bricco, *located directly across the street from the hotel.*

Sunday morning: 9:30 AM Pilar Rueda ("The approximation property and Lipschitz mappings")
+/- 10:30 AM Coffee
+/- 11:00 AM Manuel Maestre ("Holomorphy versus analyticity in the finite and infinite dimensional setting")

*A sandwich lunch will be offered to all immediately after Manolo's talk.*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# Differentiation in finite and infinite dimensions
## Kent State University, April 7 - 9, 2017
## Abstracts

**Daniel Azagra**, Univ. Complutense de Madrid
Whitney extension theorems for convex functions.

Abstract: Let $C$ be a subset of $\mathbb{R}^n$, and $f : C \to \mathbb{R}$, $G : C \to \mathbb{R}^n$ be given functions. How can we tell whether there exists a *convex* function $F : \mathbb{R}^n \to \mathbb{R}$ of some differentiability class $\mathscr{C}$ such that $F = f$ on $C$ and $\nabla F = G$ on $C$? We will give a complete answer to this question in the cases $\mathscr{C} = C^1(\mathbb{R}^n)$ and $\mathscr{C} = C^{1,\omega}(\mathbb{R}^n)$. We will also provide some partial answers for similar problems in the cases $\mathscr{C} = C^k(\mathbb{R}^n)$, $k = 2, 3, ..., \infty$.

**Estibalitz Durand Cartagena**, UNED, Madrid
AMLE's and infinity-harmonic functions in metric measure spaces

Abstract: The aim of this informal talk is to give an overview of the theory of absolutely minimizing Lipschitz extensions (AMLE) and infinity-harmonic functions in the general setting of metric measure spaces. These notions appear naturally in connection with variational problems in $L^\infty$ and are a rich source for applications.

**Piotr Hajlasz**, Univ. of Pittsburgh
A measure and orientation preserving homeomorphism with approximate Jacobian equal -1 almost everywhere

Abstract: We construct an almost everywhere approximately differentiable, orientation and measure preserving homeomorphism of a unit n-dimensional cube onto itself, whose Jacobian is equal to -1 a.e. Moreover we prove that our homeomorphism can be uniformly approximated by orientation and measure preserving diffeomorphisms. The talk is based on my joint work with Pawel Goldstein.

**Jesus Jaramillo**, Univ. Complutense de Madrid
Inversion of non-smooth mappings in Banach spaces

Abstract: We study the invertibility of nonsmooth mappings between infinite-dimensional Banach spaces. To this end, we consider a notion of set-valued pseudo-Jacobian in this setting, which is the analogue of pseudo-Jacobian matrices introduced by Jeyakumar and Luc, and which extends the Clarke generalized Jacobian. Using this, we obtain several inversion results. In particular, we give a suitable version of the classical Hadamard integral condition for global invertibility in this context.

**Pekka Koskela**, University of Jyväskylä
Planar Sobolev extension domains

Abstract: I will give an update on the efforts towards understanding why a simply connected planar domain admits or does not admit an extension operator for an associated first order Sobolev space.

**Vladimir Peller**, Michigan State Univ.
Schatten - von Neumann properties of multiple operator integrals and Lipschitz type estimates for functions of triples of self-adjoint operators

Abstract: I am going to speak about sharp recent results on Schatten - von Neumann properties of multiple operator integrals with integrands that belong to Haagerup(like) tensor products of $L^\infty$ spaces. Then I will speak about Lipschitz type estimates for functions of triples of not necessarily commuting self-adjoint operators.

**Manuel Maestre**, Univ. de Valencia

Holomorphy versus analyticity in the finite and infinite dimensional setting

Abstract: Our talk is mostly expositive based in a book [1] that we are writing with A. Defant, D. García, and P. Sevilla, and in the paper [2].

It is well-known that in several complex variables it is equivalent for a function to be separately holomorphic, Fréchet differentiable (holomorphic) and analytic. In the infinite dimensional setting this statement is not true anymore. To connect these concepts we are going to present a proof, as will appear in [1], of the following apparently elementary fact given by Hartogs in 1906. Consider in $\mathbb{C}^2$ a sequence of $m$-homogeneous polynomials $P_m = \sum_{k=0}^{m} c_{k,m} x^k y^{m-k}$ and assume that the (double) series $\sum_{m=0}^{\infty}(\sum_{k=0}^{m} c_{k,m} x^k y^{m-k})$ is convergent for every $(x, y) \in \mathbb{D} \times \mathbb{D}$, then the series

$$\sum_{m=0}^{\infty}(\sum_{k=0}^{m} |c_{k,m} x^k y^{m-k}|)$$

is also convergent in $\mathbb{D} \times \mathbb{D}$. In the second part of this talk we will apply this two-dimensional result, together with others from [2], to study the relationship between Holomorphy versus analyticity in the infinite dimensional setting.

[1] A. Defant, D. García, M. Maestre and P. Sevilla-Péris, Dirichlet series and holomorphic functions in high dimensions, in preparation.

[2] F. Bayart, A. Defant, L. Frerick, M. Maestre and P. Sevilla-Peris, Multipliers of Dirichlet series and monomial series expansions of holomorphic functions in infinitely many variables, to appear in *Math. Annalen*, DOI 10.1007/s00208-016-1511-1.

**Nageswari Shanmugalingam**, Univ. of Cincinnati

p-Harmonic versus 1-harmonic Dirichlet problem

Abstract: In this talk I will discuss recent work on the many versions of the Dirichlet problem for 1−Laplacian (or the least gradient) in the setting of metric measure spaces, and how the solutions to these problems compare to the solutions to the Dirichlet problem for $p$−Laplacian, $1 < p < \infty$. This talk is based on recent joint work with Riikka Korte, Panu Lahti, and Xining Li.

**Patrick Rabier**, Univ. of Pittsburgh
Boundedness of functions with $p$-integrable gradients

Abstract: If $f$ is a distribution on $\mathbb{R}^N$ with a locally integrable gradient, it is well-known that $f$ is a function. Furthermore, if $\nabla f \in L^1$, then $f \in L^\infty$ if $N = 1$, but this is trivially false if $N > 1$.

This lecture will discuss the existence and nature of conditions on $\nabla f$ ensuring the boundedness of $f$ or other closely related properties. The basic result is:

(i) If $\nabla f \in L^1_{loc}$ and $|x|^{1-N} * |\nabla f| \in L^\infty$, then $f \in L^\infty$

and by-products include:

(ii) If $\nabla f \in L^{N,1}$ (Lorentz space), then $f \in L^\infty$. Since $L^{N,1} = L^1$ when $N = 1$, this generalizes the one-dimensional result.

(iii) If $\nabla f \in L^p$ with $1 \le p < N$, then $f$ is "almost" in $L^\infty$ and always a finite sum of derivatives of $L^\infty$ functions.

(iv) If $\nabla f \in L^p \cap L^q_{uloc}$ with $1 \le p < N < q$, where $L^q_{uloc}$ is a space between $L^q$ and $L^q_{loc}$, then $f \in L^\infty$.

Furthermore, if $\nabla f \in L^p$ with $1 \le p < N$, then $g * f \in L^\infty$ for a fairly large class of functions $g$ that includes the Bessel kernels $g_\alpha$ for suitable (real) values of $\alpha > 0$. This can immediately be translated into the boundedness of the solution $u \in \mathscr{S}'$ (tempered distributions) of $(I - \Delta)^{\alpha/2} u = f$. When $\alpha/2$ is an integer, such results can also be obtained by the classical elliptic theory, but only when $p > 1$.

**Pilar Rueda**, Univ. de Valencia
The approximation property and Lipschitz mappings

Abstract: The importance of the approximation property in Functional Analysis has brought many authors to consider variants in nonlinear contexts. In particular, Lipschitz functions have been considered in the setting of metric spaces. However, none of these works determine specific definitions of approximation properties on metric spaces. After recalling some classical essentials regarding the approximation property, we establish a framework related to ideals of Lipschitz mappings where several options can be considered when trying to study intrinsic approximation properties for metric spaces and Lipschitz mappings. This is based on a joint work with D. Achour, E.A. Sánchez-Pérez and R. Yahi.

# Posters

Poster session during 12 - 2 PM on Saturday

**Pablo Jiménez-Rodríguez** (with R. M. Aron), Kent State University
The property of compactness under the theory of Interpolation of spaces: from operators to polynomials

Abstract: Interpolation Theory gives techniques for constructing spaces from two initial Banach spaces. We generalize some classical theorems of compact operators, to compact polynomials.

**Lukas Maly**, University of Cincinnati
Besov Spaces and Trace Classes of Sobolev Functions in Metric Spaces

Abstract: In the metric setting, one can introduce a notion of functions of fractional smoothness by adapting the definition of Besov spaces via modulus of continuity. Similarly as in $\mathbb{R}^n$, one can investigate how Besov spaces of different smoothness embed into each other and how they relate to other function spaces. The distinctive advantage of this particular approach to Besov classes over a doubling metric measure space is that most of the results can be easily proven by elementary methods.

It is also possible to show that Besov functions arise naturally as the boundary traces of Sobolev-type functions defined in a domain in a (locally) complete doubling metric measure space. The exact parameters of the Besov class obtained as a trace depend on regularity of the domain and its boundary, and on the particular choice of a Sobolev-type space. In fact, if a John domain admits a $p$-Poincaré inequality, then the trace operator maps the Newton-Sobolev space $N^{1,p}(\Omega)$ onto a certain Besov class and there is a bounded linear right inverse operator, provided that $p$ exceeds the Hausdorff codimension of the domain's boundary.

**Ben Mackey**, Michigan State University
Multipoint estimates for radial and whole plane SLE

From the Introduction: Radial $SLE_\kappa$ is a type of random fractal curve, usually defined in the unit disc $D = \{z \in \mathbb{C} : |z| < 1\}$ which grows from the boundary point 1 to the interior point 0. The behavior of the curve $\gamma : [0, \infty) \to \overline{D}$ depends on $\kappa$, and the dimension is $d = \min\{1 + \kappa/8, 2\}$. Whole plane $SLE_\kappa$ curve $\gamma^* : (-\infty, \infty) \to \mathbb{C}$ can be seen as a limit of radial $SLE_\kappa$ in a large disc, and grows from 0 to $\infty$.

**Tanmoy Paul**, Indian Institute of Technology Hyderabad, India
Some recent development on proximinality in Banach spaces

Abstract: The aim of this talk is to discuss the behavior of the notion proximinality (existence of best approximation) for few cases; viz. the transitivity of this notion or its variant through the subspaces, duality between *the intersection properties of balls* and proximinality, stability of various strengthenings of proximinality in function spaces and generalization of this property to *Chebyshev centre* for a closed convex set. This talk is based on the following articles.

T. Paul *Various notions of proximinality in spaces of Bochner integrable functions* Adv. Operator Theory, **2**, no 1, (2017) 59–77.
S. Lalithambigai et al *Chebyshev centres and some geometry of Banach spaces*, J. Math. Anal. Appl., **499**, no 1, (2017) 926–938.
C. R. Jayanarayanan et al *Strong proximinality and intersection properties of balls in Banach spaces*, J. Math. Anal. Appl., **426**, no 2, (2015), 1217–1231.