Problem 1. Let:
\[
\begin{pmatrix}
-2 & 0 & -1 & 2 \\
6 & 2 & -4 & 4 \\
0 & 0 & -8 & 1 \\
-3 & 0 & 0 & 0
\end{pmatrix}
\]
Find: \(\det(A), \det(A^2), \det(A^{-1}), \det((A^T)^2)\).

Problem 2. Find the inverse of the given matrix,
\[
A = \begin{bmatrix}
0 & 1 & 1 \\
1 & 1 & 0 \\
1 & 0 & 1
\end{bmatrix}.
\]

Problem 3. \(W\) is the set of all \(3 \times 3\) matrices of the form
\[
\begin{pmatrix}
a & 2a & 0 \\
-a & a & -3a \\
a^2 & 0 & 0
\end{pmatrix}.
\]
Is it true that \(W\) is a subspace of \(M_{3,3}\)

Problem 4. Determine whether the set \(W = \{(x, -y, x+y) : x, y \text{ are real numbers}\}\) is a subspace of \(\mathbb{R}^3\) with the standard operations.

Problem 5. Check if the set \(S = \{(0, 1, 2), (1, 1, 0), (1, -3, 2)\}\) spans \(\mathbb{R}^3\).

Problem 6. If it is possible, write vector \(w = (1, 3, 1)\) as a linear combination of vectors \((1, 1, 2), (1, 1, 0), (1, 0, 1)\).

Problem 7. Determine if the set of vectors
\[
S = \{1 + x + x^2, x^2 - 2x, 1 + 2x - 2x^2\}
\]
is linearly dependent.

Problem 8. Show that \([-2, x - 1, -x^2 + x]\) is a basis of \(P_2\).

Problem 9. Consider matrix \(A\) such that
\[
A = \begin{bmatrix}
-11 & -2 & 0 & 1 & 0 \\
0 & 1 & 1 & -1 & -1 \\
-2 & 1 & 2 & -1 & 0
\end{bmatrix}.
\]
Find
- Basis and dimension of the row space of \(A\).
• Basis and dimension of the column space of $A$.
• $\text{rank}(A)$.
• Null set of $A$.
• $\text{nullity}(A)$.

**Problem 10.** Determine whether the nonhomogeneous system of equations is consistent. If yes, then find all solutions (i.e. write the solution in the form $x = x_h + x_p$, where $x_h$ is a solution of corresponding homogenous system and $x_p$ is a particular solution).

\[
\begin{align*}
    x_1 + x_2 + x_3 + x_4 &= 1 \\
    -x_1 + 3x_2 - x_3 + 2x_4 &= -1 \\
    x_1 - 2x_2 + x_3 + 3x_4 &= 1
\end{align*}
\]

**Problem 11.** Find the transition matrix from basis $B$ to basis $B'$ if

$B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$
and

$B = \{(1, 2, -1), (-1, 0, 1), (0, 1, 1)\}$.

Also find $[x]_{B'}$ if $[x]_B = (-1, 2, -3)$.

**Problem 12.** Please, find eigenvectors and eigenvalues of the following matrix.

\[
A = \begin{bmatrix}
    3 & 2 & 1 \\
    0 & -1 & 0 \\
    0 & 3 & 3
\end{bmatrix}
\]